Registration No : | | | | | | | | | | |

Total Number of Pages : 02 **M.Tech.**
P2CTCC10

2nd Semester Regular / Back Examination 2017-18
GRAPH THEORY
BRANCH : COMPUTER ENGG,
COMPUTER SCIENCE, COMPUTER SCIENCE AND ENGG, COMPUTER SCIENCE AND TECH., INFORMATION TECH. ENGG, INFORMATION TECH.
Time : 3 Hours
Max Marks : 100
Q.CODE : C961

Answer Question No.1 which is compulsory and any FOUR from the rest.
The figures in the right hand margin indicate marks.
Answer all parts of a question at a place.

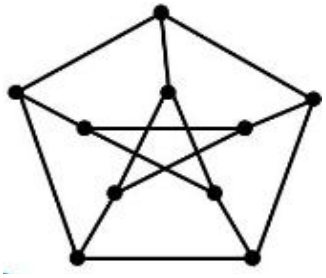
Q1 **Answer the following questions: *multiple type or dash fill up type:*** **(2 x 10)**

a) The number of vertices with odd degree is always _____ .
b) A vertex with minimum eccentricity in a graph (G) is called _____ of G.
c) A graph in which all vertices are of equal degree, is called a _____ graph.
d) Removal of any one edge from a Hamiltonian circuit generates a _____ path.
e) A graph is a _____ if and only if it is minimally connected.
f) A graph G has a _____ if and only if q(G-S) <= |S| for all S ⊆ V (G)
g) A _____ bipartite graph has a perfect matching.
h) A non-empty connected graph G is Eulerian, if all of its vertices are of _____ degree.
i) Every maximal planar graph (n ≥ 3) is _____ vertex-connected.
j) Minimum number of edges whose removal makes a Connected graph G, disconnected is called as _____ of G.

Q2 **a)** If Graph G has a Hamilton path, prove that W(G-S) <= |S|+1 for every proper subset S of V. **(10)**

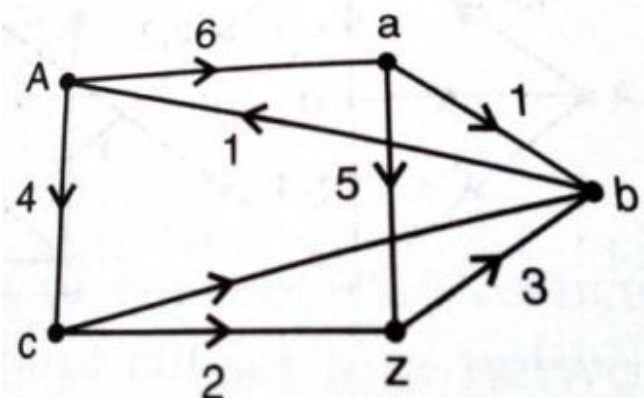
b) Define Cut-set matrix and Fundamental cut-set matrix with example of each **(10)**

Q3 **a)** Define the planar Graph and prove that the following Petersen graph is nonplanar using Kuratowski's theorem. **(10)**



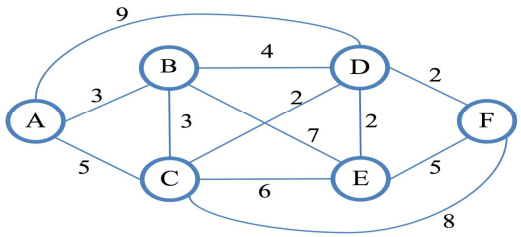
b) Prove that the set consisting of all the circuits and the edge-disjoint union of circuits (including the null set) in a graph G is an abelian group under the ring sum operation. **(10)**

Q4 **a)** Verify Max-flow Min-cut theorem for the following transport network: **(10)**



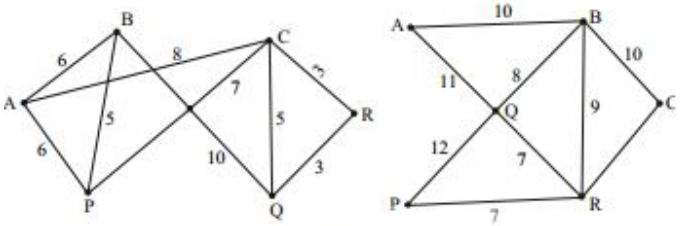
b) Prove that in a transport network G the value of flow from source S to sink D is less than or equal to the capacity of any cut that separates S from D. **(10)**

Q5 **a)** Describe Dijkstra's algorithm for determining the shortest path between two specified vertices in a connected weighted graph. Using Dijkstra's algorithm, find the shortest path from A to F in the weighted graph G of the following figure. **(10)**



b) If two graphs G_1 and G_2 are 1-isomorphic, prove that the Rank of G_1 equals the rank of G_2 and Nullity of G_1 equals the nullity of G_2. **(10)**

Q6 **a)** What do you mean by minimum spanning tree of a weighted graph? Using the kruskal's algorithm, Find a minimum spanning tree of the weighted graphs(G1 & G2) given in following figure: **(10)**



(G1) (G2)

b) Prove that covering 'h' of a graph G is minimal if and only if 'h' contains no path of length three or more. **(10)**

Q7 **a)** Define edge connectivity and vertex connectivity of a graph. Construct a graph G with edge
connectivity 4, vertex connectivity 3 and degree of each vertex of G ≥ 3. **(10)**

b) Show that the Chromatic polynomial of a graph of n vertices satisfies inequality: **(10)**

$$P_n(\lambda) <= \lambda(\lambda-1)^{n-1}$$