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# $2^{\text {nd }}$ Sem Mtech Regular/ Back Examination - 2015-16 STRUCTURAL OPTIMISATION BRANCH(S): STRUCTURAL ENGINEERING <br> Time: 3 Hours <br> Max marks: 70 <br> Q.CODE:W777 

## Answer Question No. 1 which is compulsory and any five from the rest. The figures in the right hand margin indicate marks.

Q1 Answer the following questions:
a) Define a local and global maximum.
b) State a linear and non-linear optimization problem.
c) Write the condition for linear independence of a set of vector $a_{1}, a 2, \ldots, a_{k}$.
d) When a function is called concave and convex function.
e) Determine whether the following junction is convex or concave $f(x)=-2 x^{2}+8 x+4$.
f) Determine whether the following function is positive definite or, negative definite or neither, $f(x)=-x_{1}{ }^{2}+4 x_{1} x_{2}+4 x_{2}{ }^{2}$.
g) Define Lagrange function and language multiplier.
h) What are the necessary and sufficient conditions for a stationary point $X$ to be minimum?
i) When a point is called saddle point. Give an example.
j) Write down the characteristic of linear programming.

Q2 a) State Lagrangian method to find an optimum of function of $n$ variables subject to $m$ $(m \leq n)$ constraints.
b) Obtain optimum solution of the following non-linear programming problem,

$$
\begin{equation*}
\operatorname{Maxf}(\mathrm{x})=2 \mathrm{x}_{1}+\mathrm{x}_{2}+10 \tag{5}
\end{equation*}
$$

Subject of constraints;
$X_{1}+2 X_{2}=3$
$\mathrm{X}_{1}, \mathrm{X}_{2} \geq 0$
Q3 a) Find the dimension of a cylindrical tin (with top and bottom) made up of sheet metal
to maximize its volume such that the total surface area is equal to $A_{0}=24 \pi$.
Q4 Use the Wolfe's method to solve the given quadratic programming problem

$$
\begin{equation*}
\text { Maximize } z=4 x_{1}+6 x_{2}-2 x_{1}^{2}-2 x_{1} x_{2}-2 x_{2}^{2} \tag{10}
\end{equation*}
$$

Subjected to constraints
$\mathrm{X}_{1}+2 \mathrm{X}_{2} \leq 2$ and
$\mathrm{X}_{1}, \mathrm{X}_{2} \geq 0$.
Q5 a) State when the function $f(x)$ is positive definite and negative definite or saddle point.
b) Find the maximum or minimum of function $f(x)=x_{1}+2 x_{3}+x_{2} x_{3}-x_{1}{ }^{2}-x_{1}{ }^{2}-x_{2}{ }^{2}-x_{3}{ }^{2}$.

Q6
a) Write quadratic form of a function
$Q(x)=\left(x_{1}, x_{2}, x_{3}\right)^{\top}$ and matrix

$$
A=\left[\begin{array}{lll}
1 & 0 & 1  \tag{5}\\
2 & 7 & 6 \\
3 & 0 & 2
\end{array}\right]
$$

Why the matrix A can be assumed to be symmetric?
b) Write down when a quadratic form is said to be

1) Positive definite,
2) Positive semi-definite,
3) Negative definite,
4) Negative semi-definite,
5) In-definite

Q7 a) What are necessary as well as sufficient Kuhn-Tucker conditions for obtaining an absolute (or global) maximum of $\mathrm{f}(\bar{X})$ at $\bar{X}$ ? Also
Maximize $Z=12 x_{1}+21 x_{2}+2 x_{1} x_{2}-2 x_{1}{ }^{2}-2 x_{2}{ }^{2}$
Subjected to constraints

1) $X_{2} \leq 8$
2) $X_{1}+X_{2} \leq 10$.

For the rigid frame shown in fig. 1, plastic moments may develop at the point of peak moments (numbered 1 through 7 ). Assuming that the weight is a linear function of plastic moment capacity of beam $\left(M_{b}\right)$ and column $\left(\mathrm{M}_{\mathrm{c}}\right)$, form the optimization statement under minimum weight condition. Take $w=W$ eight/unit length/moment. For the structure to be safe, the energy absorbing capacity of the frame $(U)>$ the the energy imparted by externally applied load ( E ) for various collapse mechanisms. $P_{1}=3, P_{2}=1, h=8, l=10$


