

Registration no:

--	--	--	--	--	--	--	--	--	--

Total Number of Pages: 02

M.Sc.I  
FMCC7017<sup>th</sup> Semester Regular Examination 2017-18

Topology

Branch: M.Sc.I (MC)

Time: 3 Hours

Max Marks: 70

Q. CODE: B625

Answer Question No.1 which is compulsory and any five from the rest.

The figures in the right hand margin indicate marks.

Q1 Answer the following questions: (2 x 10)

- Define Subspace and Hereditary Property.
- Define limit point and derived set.
- Consider  $X = \{x, y, z\}$ ,  $\mathcal{T} = \{\emptyset, \{x\}, \{y, z\}, X\}$ ,  $Y = \{p, q, r\}$ ,  $\mathcal{V} = \{\emptyset, \{r\}, \{p, q\}, Y\}$ .  $(X, \mathcal{T})$  and  $(Y, \mathcal{V})$  are two topological spaces. Consider the map  $f$ . (i)  $f(x) = r$ ,  $f(y) = p$ ,  $f(z) = q$  discuss continuity and openness.
- Define 1<sup>st</sup> and 2<sup>nd</sup> countable base.
- Define Homeomorphism.
- Define Continuity with an example.
- Define open map, closed map, define topological property.
- Define Hausdorff space with an example.
- Define locally connected set with an example
- Define Compactness.

Q2 a) Let  $X$  be a non-empty set  $\mathcal{F}$  a family of subsets of  $X$  such that (5)

$$[\mathcal{F}:1] : \emptyset \in \mathcal{F}, X \in \mathcal{F}$$

$$[\mathcal{F}:2] : F_1 \in \mathcal{F}, F_2 \in \mathcal{F} \Rightarrow (F_1 \cup F_2) \in \mathcal{F}$$

$$[\mathcal{F}:3] : F_i \in \mathcal{F}, \text{ for all } i \in I \Rightarrow \bigcap F_i \in \mathcal{F} \text{ for all } i \in I$$

Then there is an unique topology  $\mathcal{T}$  on  $X$  s. t. the  $\mathcal{T}$  closed subsets are precisely the members of  $\mathcal{F}$ .

- Let  $(X, \mathcal{T})$  be a topological space and let  $A$  be a subset of  $X$ . the set  $A$  is  $\mathcal{T}$  - open iff  $A$  contains a  $\mathcal{T}$  - neighborhood of each of its points. (5)

Q3 a) The  $(X, \mathcal{T})$  be a topological space and let  $A \subset X$ . Then  $A$  is a  $\mathcal{T}$ -closed iff  $A' \subset A$ . (5)

- Let  $(Y, \mathcal{V})$  be a subspace of  $(X, \mathcal{T})$ . If  $(Z, \mathcal{W})$  is a subspace of  $(Y, \mathcal{V})$  then  $(Z, \mathcal{W})$  is also a subspace of  $(X, \mathcal{T})$ . (5)

Q4 a) Let  $X, X^*, X^{**}$  be topological spaces and let  $f : X \rightarrow X^*$ , and  $g : X^* \rightarrow X^{**}$  (5)

Be continuous. Show that the composition map  $g \circ f : X \rightarrow X^{**}$

- b)** Consider the topological space  $(X, \mathcal{T})$  where  $X = \{x, y, z\}$  and  $\mathcal{T} = \{ \emptyset, \{x\}, \{x, y\}, \{x, z\}, X \}$  Find the derived sets of  $A = \{y, z\}$  and  $B = \{z, x\}$ . **(5)**
- Q5 a)** Let  $(X, \mathcal{T})$  and  $(Y, \mathcal{V})$  be two topological spaces. A mapping  $f : X \rightarrow Y$  is continuous iff for every  $\mathcal{V}$ -closed set  $F$ ,  $f^{-1}[F]$  is  $\mathcal{T}$ -closed. **(5)**
- b)** Let  $(X, \mathcal{T})$  and  $(Y, \mathcal{V})$  be two topological spaces. A mapping  $f$  from  $X$  to  $Y$  is  $\mathcal{T}$ - $\mathcal{V}$  closed iff  $f[A] \subset f[\overline{A}]$  for every  $A \subset X$ . **(5)**
- Q6 a)** Show that homeomorphism is an equivalence relation in collection of all topological space. **(5)**
- b)** A topological space  $(X, \mathcal{T})$  is connected iff it has no non-empty proper subsets which are both  $\mathcal{T}$ -open and  $\mathcal{T}$ -closed. **(5)**
- Q7 a)** If  $C$  is connected set and  $C \subset E \subset \overline{C}$  then  $E$  is connected set. **(5)**
- b)** If  $E$  is a subset of a subspace  $(Y, \mathcal{T}_y)$  of a topological space  $(X, \mathcal{T})$  then  $E$  is  $\mathcal{T}_y$ -connected set iff  $E$  is  $\mathcal{T}$ -connected. **(5)**
- Q8 a)** Let  $(Y, \mathcal{T}_y)$  be a subspace of a topological space  $(X, \mathcal{T})$  and let  $A \subset Y$  then  $A$  is  $\mathcal{T}$ -compact iff  $A$  is  $\mathcal{T}_y$ -compact. **(5)**
- b)** Continuous image of compact sets are compact. **(5)**