Registration No :

Total Number of Pages : 02 **M.Sc.I**
FMCC702

7th Semester Regular Examination 2019-20
MEASURE THEORY
BRANCH : M.Sc.I(MC)
Time : 3 Hours
Max Marks : 70
Q.CODE : HR180
Answer Question No.1 which is compulsory and any FIVE from the rest.
The figures in the right hand margin indicate marks.

Q1 **Answer the following questions :** **(2 x 10)**

a) If x is real then find $(m^*[x])$.

b) If F is a measurable set and $m^*(F\Delta G) = 0$ then show that G is measurable.

c) If $\{E_i\}$ be a sequence of measurable sets and $E_1 \subseteq E_2 \subseteq \cdots$ then show that

$$m\left(\lim_i E_i\right) = \lim_i m(E_i)$$

d) Show that if f is integrable then f is finite-valued a.e.

e) Is every measurable set a Borel set? Give reason.

f) If A be any set and $\{E_k\}_{k=1}^n$ a finite disjoint collection of measurable sets then show that $m^*(A \cap [\cup_{k=1}^n E_k]) = \sum_{k=1}^n m^*(A \cap E_k)$.

g) If f is non-negative measurable function then show that $f = 0$ a.e. if and only if $\int f \ dx = 0$.

h) Show that the Lebesgue set of a function $f \in L(a, b)$ contains any point at which f is continuous.

i) Show that $\int_1^\infty \frac{dx}{x} = \infty$.

j) If $\emptyset$ is a measurable simple function then show that $\int_{\mathbb{R}} \emptyset \, dx = \sum_{i=1}^n a_i \, m(A_i \cap E)$ for any measurable set E.

Q2 **a)** Show that for any set A, $m^*(A) = m^*(A + x)$ where $A + x = \{y + x : y \in A\}$ **(5)**

b) Show that the outer measure of an interval equals its length. **(5)**

Q3 **a)** Show that the class of all measurable sets is a $\sigma -$ algebra. **(5)**

b) Suppose that f is any extended real-valued function which for every x and y satisfies $f(x) + f(y) = f(x + y)$. Show that if f is measurable and finite then $f(x) = xf(1)$ for each x. **(5)**

Q4 **a)** Show the following statements regarding the set E are equivalent: **(5)**

i) E is measurable,

ii) for all $\varepsilon > 0$, there exist A, an open set, $A \subseteq E$ such that $m^*(A - E) \leq \varepsilon$.

iii) there exist G, a G_δ set, $G \supseteq E$ such that $m^*(G - E) = 0$.

b) State and prove Lebesgue's dominated convergence theorem. **(5)**

Q5	**a)**	Is every interval measurable? Give reason.	**(5)**
	b)	Show that there exist uncountable sets of zero measure.	**(5)**
Q6	**a)**	Are continuous functions measurable? Give reason.	**(5)**
	b)	Show that $f \leq ess \sup f$, a.e.	**(5)**
Q7	**a)**	Find $\lim \int_0^\infty \frac{dx}{(1+\frac{x}{n})^n x^{1/n}}$.	**(5)**
	b)	Show that a function $f \in BV[a,b]$ if and only if f is the difference of two	**(5)**
Q8		**Write short Notes on :**	
	a)	Find $\lim \int_a^\infty \frac{n^2 x e^{-n^2 x^2}}{1+x^2}\, dx$.	**(5)**
	b)	State and prove Lebesgue's differentiation theorem.	**(5)**