Registration No: | | | | | | | | | | |

Total Number of Pages: 02 **M.Sc.I**
FMCC703

7th Semester Regular Examination 2017-18
Advanced Differential Equation
BRANCH : M.Sc.I(MC)
Time: 3 Hours
Max Marks: 70
Q.CODE: B627
Answer Question No.1 which is compulsory and any five from the rest.
The figures in the right hand margin indicate marks.

Q1 **Answer the following questions :** **(2 x 10)**

a) If the half-life period of radium is 1600 years, how long it take for 10% of the original amount of material to disintegrate?

b) Make a mathematical model for a Resistor, Inductor and capacitor circuit to determine current flow.

c) Find $H_3(x)$ *and* $H_4(x)$ *using* Hermite polynomial.

d) Represent the following in terms of Hermite polynomial

$$f(x) = 1 + x + x^2.$$

e) Find the indicial equation of the differential equation

$$x^2\, y^{II} + 4xy^{I} + (x^2 + 2) = 0.$$

f) If $\frac{dx}{dt} = a_{11}(t)\, x + a_{12}(t)y + f_1(t)$

$$\frac{dy}{dt} = a_{21}(t)\, x + a_{22}(t)y + f_2(t)$$

then write the general solution of the above system

g) Write D'Alembert's solution of one dimensional wave equation.

h) Write the physical assumptions for two dimensional wave equation.

i) Write Green's function of the Dirchlet problem for Laplace equation.

j) Test whether the differential equation is parabolic, hyperbolic or Elliptic

$$u_{xx} - 4u_{xy} + 4u_{yy} = 0\ , \ \ where\ u = u(x, y)$$

Q2 **Answer the questions :**

a) Find the current in the simple circuit with C=∞ and E(t)=$E_0 \sin \omega t$. **(5)**

b) In a chemical reaction the amount x of a substance at time t satisfies **(5)**
$\frac{dx}{dt} = k(3 - x)(6 - x)$, where x *is constant. If*
$x = 0$, *when* $t = 0$, *and* $x = 1$ *when* $t = 10$, *find the value of* k
What is the value of x *when* $t = 30$?

Q3 **a)** Show that $H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n}\, e^{-x^2}$. **(5)**

b) Using the method of Frobenius to find solution near $x = 0$ of the differential equation **(5)**

$$2x^2\, y^{II} + xy^{I} + (x^2 - 1) = 0.$$

Q4 Using operator method solve the system of differential equation **(10)**

$$\frac{d^2x}{dt^2} + \frac{dy}{dt} = e^{2t}$$
$$\frac{dx}{dt} + \frac{dy}{dt} - x - y = 0.$$

Q5 Using matrix method solve the system of differential equation **(10)**

$$\frac{dx}{dt} = -14\,x + 10y$$
$$\frac{dy}{dt} = -5\,x \;+\; y$$
$$x(0) = -1\,,\; y(0) = 1$$

Q6 A taut string of length l has its ends $x = 0, and\ x = l\ \ fixed.$ The point where [10] $x = \frac{l}{3}\ \ is$ drawn aside a small distance h and released at time t=0 . At any subsequent time t > 0 the displacement $y(x,t)$ satisfies cone dimensional wave equation **(10)**

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}.$$

Determine $y(x,t)\ at\ any\ time\ t$

Q7 Find the steady temperature distribution $u(x,y) in\ the\ uniform$ unit square $0 \le x \le 1$; $0 \le y \le 1$; when the edge y = 1, is maintained at the temperature$x(1-x)$, the other three edges being thermally insulated so that $\frac{\partial u}{\partial n}\ along\ them$. **(10)**

Q8 Solve the following initial boundary value problem for the transient temperature $u(x,y,t)$ for the diffusion of heat in a rectangular plate of uniform, isotropic material **(10)**

$$u_{xx} + u_{yy} = \frac{1}{k}u_t \qquad 0 < x < a\,, 0 < y < b\,, t > 0$$
$$u(x,0,t) = u(x,b,t) = 0 \qquad 0 < x < a\,, t > 0$$
$$u(0,y,t) = u(a,y,t) = 0 \qquad 0 < y < b\,, t > 0$$
$$u(x,y,t) = f(x,y) \qquad 0 < x < a\,, 0 < y < b$$