Registration no: | | | | | | | | | | |

Total Number of Pages: 03 **M.Sc.I**
FMCC603

6^{TH} Semester Regular / Back Examination – 2017-18
DIFFERENTIAL EQUATION-II
BRANCH: , M.Sc.I(MC)
Time: 3 Hours
Max marks: 70
Q CODE : C284
Answer Question No.1 which is compulsory and any five from the rest.
The figures in the right hand margin indicate marks.

Q1 **Answer the following questions:** **(2 x 10)**

a) Write the definition of boundary value problem and Strum-Liouville problem.

b) Show that the set of functions $\varphi_n(x) = \sin nx$, n=1,2,3,4,….. on [0,π]with respect to the weight function 1 on [0,π]is orthogonal.

c) show that Green's function is symmetric in nature.

d) Find the partial differential equation from the equation $z = e^{mx} f(x,y)$.

e) Find the solution of Clairaut equation $z = px + qy + f(p,q)$.

f) Write the kind of solution of the homogenous linear reducible partial differential equation with constant coefficients $F(D, D')z = 0.$

g) Find the particular integral of $(D^2 - D')z = e^y, where\ z = z(x,y)$

h) Using direct integration method to find the solution of $\frac{\partial u}{\partial x} = xyu$,

$u = u(x,y).$

i) Test for elliptic, parabolic and hyperbolic condition of the differential equation $u_t = 4u_{xx}$,$where\ u = u(x,t).$

j) Write the subsidiary equations of Monge's method for the partial differential equation
$Rr + Ss + Tt = V,\ where\ r = \frac{\partial^2 u}{\partial x^2},\ \ s = \frac{\partial^2 u}{\partial x \partial y},\ t = \frac{\partial^2 u}{\partial y^2}\ \ and\ u = u(x,y)$

Q2 a) Solve the boundary value problem **(5)**

$y'' + 4y = f(x)$, $y(0) = 0$ *and* $y'(\pi) = 0$ using appropriate Green's function where f(x) = x^2.

b) Find the Eigen values and Eigen functions of the following Sturm-Liuville problem **(5)**
$\frac{d}{dx}\left(x\frac{d}{dx}\right) + \frac{k}{x}y = 0$, $y(1) = 0$ *and* $y(e^{\pi}) = 0$

Q3 a) Verify that the differential equation **(5)**

$2yzdx - 2xzdy - (x^2 + y^2)(z-1)dz = 0$ is integrable and then find the solution.

b) Solve the differential equation $\frac{dx}{x(y^2+z)} = \frac{dy}{-y(x^2+z)} = \frac{dz}{z(x^2-y^2)}$ **(5)**

Q4 a) If $\psi_m(x)$ *and* $\psi_n(x)$ *are eigen functions* of the Sturm-Liouville problem corresponding to the distinct Eigen values λ_m *and* λ_n *respectively,* then prove that they are orthogonal with respect to the weight function γ(x). **(5)**

b) Find the complete integral of the differential equation $f_1(x,p) = f_2(y,q)$ where $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$ and $z = z(x,y)$. **(5)**

Q5 a) Find the general solution of the partial differential equation **(5)**

$zxp - zyq = y^2 - x^2$, where $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$ *and* $z = z(x,y)$.

b) Find the complete integral of the equation **(5)**

$(p^2 - q^2)y = qz$ *by Charpit's method.*

Q6 a) Prove that If two functions u(x,y) and v(x,y) satisfy the relation $\frac{\partial(u,v)}{\partial(x,y)}$ =0, then there exists between them a relation F(u,v)=0,not involving x or y explicitly. **(5)**

b) Find the general solution of $(D^2 + DD' + D + D' + 1)z = e^{-2x}(x^2 + y^2)$. **(5)**

Q7 a) If $\{a_r D + b_r D' + c_r\}^2$, $\{for\ a_r \neq 0\}$ is a factor of $F(D, D')$ and if $\phi(x,y)$ and $\psi(x,y)$are arbitrary functions , then show that **(5)**

$e^{-\frac{c_r}{a_r}x}\{x\phi_r(b_r x - a_r y) + \psi_r(b_r x - a_r y)\}$ is a solution of $F(D, D')z = 0$, where z = z(x,y).

b) Solve the differential equation **(5)**

$$\left(x^2D^2 - xyDD' - 2y^2D'^2 + xD - 2yD'\right)z = ln\frac{x}{y}.$$

Q8 a) Using product method solve $\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = 0$, *where* $u = u(x,y)$. **(5)**

b) Using Monge's method solve the partial differential equation **(5)**

$x^2r + 2xys + y^2t = 0$, $r = \frac{\partial^2 u}{\partial x^2}, s = \frac{\partial^2 u}{\partial x \partial y}, t = \frac{\partial^2 u}{\partial y^2}$ *and* $u = u(x,y)$.