Registration no: | | | | | | | | | | |

Total Number of Pages: 02 **M.Sc.I**
FPYC602

6th SEMESTER REGULAR/BACK EXAMINATION – 2017-18
MATHEMATICAL METHODS -II
BRANCH(S): M.Sc.I(AP)
Time: 3 Hours
Max marks: 70
Q.CODE:C236

Answer Question No.1 which is compulsory and any five from the rest.
The figures in the right hand margin indicate marks.

Q1 **Answer the following questions:** **(2 x 10)**

a) Evaluate $\int_{-1}^{1}(1+X)^{P-1}(1-X)^{q-1}dx$.

b) Show that $\int_0^{\frac{\pi}{2}}\tan^p\theta\, d\theta = \frac{\pi}{2}\sec p\pi/2$.

c) Find the complex form of Fourier series of f(x)=x on the interval $-\pi$ <x< π

d) Expand f(x) = k for 0 < x< 2 in a half range sine series.

e) Prove the recurrence relation $np_n(x) = xp_n'(x) - p_{n-1}'(x)$

f) Distinguish between systematic and Random error.

g) What do u mean by Normal law of error?

h) Prove that $H_n(-x) = (-1)^n H_n(x)$

i) Applying method of separation of variable techniques, find the solution to P.D.E

$3u_x + 2u_y = 0$, where $u_x = \partial u/\partial x$, $u_y = \partial u/\partial y$.

j) Determine the nature of the point x= 0 for the differential equation 2xy"+(3-x) y' –y =0

Q2 **a)** Using Frobenius method ,obtain a series solution in powers of x for differential equation **(7)**
$2x^2\partial^2y/\partial x^2 + x\,\partial y/\partial x - (x+1)y = 0$

b) Find the regular singular point of the differential equation **(3)**
(3x+1)xy" –(x+1)y' +2y=0

Q3 **a)** Expand $f(x)=x^4$ for $-\pi \le x \le \pi$ in a Fourier series and prove that **(7)**

i) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n^4} = 7\pi^4/720$

ii) $\sum_{n=1}^{\infty}\frac{1}{n^4} = \frac{\pi^4}{90}$

b) Determine the value of $\beta(\frac{1}{2},\frac{1}{2})$ **(3)**

Q4 **a)** Solve Legendre differential equation **(7)**
$(1-x)^2y'' - 2xy' + n(n+1)y = 0$ by power series method. Show that the solutions are Legendre polynomials.

b) Using generating function for Legendre polynomials $p_n(x)$ show that **(3)**
$(2n+1)xp_n(x) = (n+1)p_{n+1}(x) + np_{n-1}(x)$

Q5 **a)** Derive the Rodrigues's formula for Hermite polynomials. **(6)**

b) Using generating function for Hermite polynomials $H_n(x)$ show that **(4)**
$H_{n+1}(x) - 2xH_n(x) + 2n\,H_{n-1}(x) = 0$

Q6 **a)** $Prove\ that \int_0^{\pi/2} \sqrt{sin\theta}\ d\theta \times \int_0^{\pi/2} 1/\sqrt{sin\theta}\ d\theta = \pi$ **(3)**

b) $Show\ that \int_0^{\infty} e^{-x^2-2bx} dx = \frac{\sqrt{\pi}}{2} e^{b^2}[1 - \mathrm{erf}(b)]$ **(3)**

c) $Evalute \int_0^{\infty} \frac{x^a}{a^x} dx, hence\ show\ that \int_0^{\infty} x^7/7^x\ dx = 7!/(log7)^8$ **(4)**

Q7 **a)** Solve the following equation by the method of separation of variables **(5)**
$\partial^2 u/\partial x \partial t = e^{-t} cosx$ .Given that u=0,when t=0 and ∂u/∂t=0 when x=0

b) Solve Laplace's equation in problems of cylindrical symmetry by using separation variables method. **(5)**

Q8 **Write Short Notes on any two** **(5x2)**
a) Half range cosine series
b) Parseval's formula
c) Duplication formula
d) Dirichlet's condition