Registration no:

Total number of pages: 02 **M.Sc.I**

FMCC501

5th Semester Back Examination 2017-18

Advance Calculus

Branch: M.Sc.I (MC)

Time: 3 Hours

Max Marks: 70

Q.CODE: B622

Answer Question No. 1 which is compulsory and any five from the rest.

The figures in the right hand margin indicate marks.

Q1 **Multiple Choice Questions :**

a) Define error function. **(2 x 10)**

b) Prove that $\beta(m,n) = \beta(n,m)$.

c) State Legendre condition.

d) Define isoperimetric problem with example.

e) Define a linear Fredholm integral equation of first kind with example.

f) Define field of extremal.

g) Show that the function $\phi(x) = 1$ is a solution of the Fredholm integral equation

$$\phi(x) + \int_0^1 x(e^{x\xi} - 1)\phi(\xi)d\xi = e^x - x.$$

h) What is Jacobi condition and write its equation.

i) State Euler equation for a functional of the form $I[x(t), y(t)]$ with fixed boundaries.

j) Compute $\Gamma(4.5)$

Q2 Find the extremals of the functional $I[Y(x)] = \int_{x_0}^{x_1} (2xy + y'''^2)dx$. **(10)**

Q3 **a)** Define Rodrigue's formula and show that **(5)**

$P_n'(x) = xP_{n-1}'(x) + nP_{n-1}(x)$.

b) Compute $\beta(\frac{5}{2}, \frac{3}{2})$ **(5)**

Q4 Find the extremal in the isoperimetric problem of the extremum of **(10)**

$I[Y(x), Z(x)]) = \int_0^1 (y'^2 + z'^2 - 4xz' - 4z)dx$

subject to $\int_0^1 (y'^2 - xy' - z'^2)dx = 2, y(0) = 0, z(0) = 0, y(1) = 1, z(1) = 1.$

Q5 Show that the function $\phi(x) = \sin(\pi x/2)$ is a solution of the equation **(10)**

$$\emptyset(x) = \frac{1}{2}x + \frac{\pi^2}{4}\int_0^1 k(x,\xi)\,\phi(\xi)d\xi$$

where$k(x,\xi) = \frac{1}{2}x(2-\xi), 0 \le x \le \xi \ and \ \frac{1}{2}\xi(2-x),\ \xi \le x \le 1.$

Q6 Using the Raleigh-Ritz method, find the extremum of the functional $I[Y(x)] = \int_0^1 (y'^2 + y^2)dx, y(0) = 0, y(1) = 1$. **(10)**

Q7 Find the shortest distance between the parabola $y = x^2$ and the shortest line $x - y = 5$. **(10)**

Q8 With the aid of the resolvent kernel find the solution of the integral $\phi(x) = 1 + \int_0^x (\xi - x)\phi(\xi)d\xi$. **(10)**