Registration No :

Total Number of Pages : 02 **M.Sc.I**
FMCE207

2nd Semester Back Examination 2017-18
MATHEMATICS-II
BRANCH : M.Sc.I(AP)
Time : 3 Hours
Max Marks : 70
Q.CODE : C811
Answer Question No.1 which is compulsory and any five from the rest.
The figures in the right hand margin indicate marks.
Answer all parts of a question at a place.

Q1 **Answer the following questions :** **(2 x 10)**

a) Define greatest upper bound and least upper bound of a set.

b) If M and N are neighbourhoods of a point x, then show that $M \cap N$ is also a neighbourhood of 'x'.

c) If S and T are two subsets of real numbers, then show that $S \subseteq T \Rightarrow S' \subseteq T'$. Where S' and T' are derived sets of S and T respectively.

d) Find the limit inferior and superior of the sequence $\left\{ \frac{(-1)^n}{n^2}, n \in N \right\}$.

e) What is the necessary and sufficient condition for the convergence of a positive term series ?

f) Define group and give an example of a group.

g) Show that N is a normal subgroup of G iff $gNg^{-1}=N$

h) Define homomorphism from a group G to another group $\bar{G}$. What is kernel of Homomorphism.

i) Define ring. Give an example of a commutative ring.

j) If R is a ring, then for all $a, b \in R$, $a0 = 0a$. Prove?

Q2 **a)** Show that every open interval (a,b) contains a rational number. **(5)**

b) Show that the interior of a set 'S' is the largest open subset of 'S'. **(5)**

Q3 **a)** Show that a set is closed iff its complement is open. **(5)**

b) Prove that the derived set S' of a bounded infinite set $S(\subseteq R)$ has the smallest or greatest member. **(5)**

Q4 **a)** Show that every bounded sequence has a limit point. **(5)**

b) Show that the series $\sum \frac{1}{n}$ doesn't converge. **(5)**

Q5 **a)** Explain Cauchy's root test for convergence of an infinite series. **(5)**

b) Show that the series $\sum \frac{1}{n!}$ is convergent. **(5)**

Q6 **a)** If H is a non empty subset of a group G and H is closed under multiplication, then show that H is a subgroup of G. **(5)**

b) Let H and K be finite subgroups of a group G, then show that **(5)**

$$O(HK) = \frac{O(H)O(K)}{O(H \cap K)}.$$

Q7 **a)** Show that the subgroup N of a group G is normal iff every left coset of in G is a right coset of N in G. **(5)**

b) If ϕ is a homomorphism of G into $\bar{G}$, then show that (i) $\phi(e) = \bar{e}$ and (ii) $\phi(x^{-1}) = \phi(x)^{-1}$. **(5)**

Q8 **a)** Show that, the set of even integers under the usual operations of addition and multiplication is a commutative ring. **(5)**

b) Prove that a finite integral domain is a field. **(5)**