Registration no:

M.Sc
MAMC403

4th Semester Regular Examination – 2016-17
MATRIX COMPUTATION
BRANCH(S): M.SC.(MH)
Time: 3 Hours
Max Marks: 70
Q.CODE:Z1139
Answer Question No.1 which is compulsory and any five from the rest.
The figures in the right hand margin indicate marks.

Q1 **Answer the following questions:** **(2 x 10)**

a) Let G be a triangular matrix, then G is non-singular iff $g_{ii} \neq 0$ for $i = 1,2.....n$ .Prove it.

b) Define triangular matrix.

c) Define positive definite matrix and give an example.

d) Define matrix norm on R^n .

e) Show that $k(A) = k(A^{-1})$.

f) Write down the maximum and minimum magnification of matrix A.

g) Show that if U is unitary then absolute value of determinant of U is 1.

h) Show that A is non-singular iff zero is not an eigen value of A.

i) Define invariant subspace.

j) Show that $|\lambda_k| \succ |\lambda_{k+1}|$ implies that $N(A) \le U_k$.

Q2 **a)** Use the column oriented version of forward substitution to solve the triangular system $4y_1 = 12, 2y_1 - 6y_2 = -4, y_1 + 3y_2 + 5y_3 = 10$. **(5)**

b) Let M be any $n \times n$ non-singular matrix and let $A = M^T M$ .Then A is positive definite. **(5)**

Q3 **a)** Using the value of $A = \begin{bmatrix} 16 & 4 & 8 & 4 \\ 4 & 10 & 8 & 4 \\ 8 & 8 & 12 & 10 \\ 4 & 4 & 10 & 12 \end{bmatrix}$ & $B = \begin{bmatrix} 32 \\ 26 \\ 38 \\ 30 \end{bmatrix}$ and also use the inner product formulation of cholesky's method to show that A is a positive definite and compute its cholesky factor **(5)**

b) Find the general solution of $\dot{x}_1 = 2x_1 + 3x_2$ $\dot{x}_2 = x_1 + 4x_2$. **(5)**

Q4 Let A be an $n \times n$ matrix whose leading principal submatrices are all non-singular.Then A can be decomposed in exactly one way as a product $A = LDV$. **(10)**

Q5 **a)** For all $x, y \in R^n$ show that $\left|\sum_{i=1}^{n} x_i y_i\right| \le \left(\sum_{i=1}^{n} x_i^2\right)^{1/2}\left(\sum_{i=1}^{n} y_i^2\right)^{1/2}$ **(5)**

b) Let $A = \begin{bmatrix} 8 & 1 \\ -2 & 1 \end{bmatrix}$.Use power method with $q_0 = [1 \quad 1]^T$ to calculate the dominant eigen value and eigen vector of A. **(5)**

Q6 **a)** Prove that $\|A\| = \max_{1 \le j \le n} \sum_{i=1}^{n} |a_{ij}|$. **(5)**

b) Use a QR decomposition to solve the linear system **(5)**
$\begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 12 \\ 29 \end{pmatrix}$.

Q7 **a)** Let $x, y \in R^n$ with $x \ne y$ but $\|x\|_2 = \|y\|_2$.Then there is a unique reflector Q such that $Qx = y$. **(5)**

b) Show that if $U \in C^{n \times n}$ is unitary and $x, y \in C^n$,then (i) $\langle u_x, u_y \rangle = \langle x, y \rangle$ (ii) $\|u_x\|_2 = \|x\|_2$ (iii) $\|u\|_2 = \|u^{-1}\|_2 = k_2(u) = 1$. **(5)**

Q8 **a)** Suppose $q, Aq, \ldots\ldots\ldots A^{m-1}q$ are linearly independent .Then $K_m(A_1 q)$ is invariant under A iff $q, Aq, \ldots\ldots\ldots A^{m-1}q, A^m q$ are linearly dependent. **(5)**

b) Let S be subspace of F^n with a basis $x_1, x_2, \ldots\ldots x_k$ .Then $S = span\{x_1.x_2, \ldots\ldots x_k\}$.Let $\hat{X} = [x_1, x_2, \ldots\ldots x_k] \in F^{n \times k}$.Then S is invariant under $A \in F^{n \times n}$ iff there exist $\hat{B} \in F^{k \times n}$ such that $A\hat{X} = \hat{\mathbf{B}}\hat{X}$. **(5)**