Registration No : | | | | | | | | | | |

Total Number of Pages : 02 **M.Sc.**

15MMCC203

2^{nd} Semester Back Examination 2017-18
COMPLEX ANALYSIS
BRANCH : M.Sc.(MC), M.SC.(MH)
Time : 3 Hours
Max Marks : 70
Q.CODE : C805
Answer Question No.1 which is compulsory and any five from the rest.
The figures in the right hand margin indicate marks.
Answer all parts of a question at a place.

Q1 **Answer the following questions :** **(2 x 10)**

a) Verify Cauchy Riemann equation f(z)=ln $|z|$

b) What is bilinear Transformation? Give one example.

c) Find co-efficient of $(z-\pi)^2$ in the Taylor's series expansion of f(z) =$\{\frac{\sin z}{z-\pi}$, if $z \neq \pi$
-1 , if z= π around of z.

d) If f:c→ c be analytic except for a simple pole at z=0 and g: c→ c be analytic, then find the value of $\left\{\frac{\underset{z=0}{Res}\{f(z).g(z)}{\underset{z=0}{Res}f(z)}\right.$

e) What is the principal value of log($i^{\frac{1}{4}}$) ?

f) Define essential singularities and give an example?

g) Define Roucher's Theorem? Give one example.

h) For the function f(z)= sin($\frac{1}{cos\frac{1}{z}}$) at the point z=0 is which singularity.

i) If f(z)= $\sum_{n=0}^{15} z^n$ for z ∈ c, if c: $|$z-i$|$ =2 then find the value of $\oint \frac{f(z)}{(z-i)^{15}}$dz

j) State Morera's Theorem?

Q2 **a)** Prove that an entire bounded function in a complex plane is constant. **(5)**

b) Find the linear fractional transformation which maps i, 0, 1 onto 2+I ,2 ,3 **(5)**

Q3 **a)** State and prove the Cauchy residue Theorem? **(5)**

b) $\int_c \bar{z}$ dz ,c from 0 along the parabola y=x^2 to 1+i **(5)**

Q4 **a)** Suppose that f is analytic in bounded domain D and continuous on $\bar{D}$. Then prove that $|$f(z)$|$ attain its maximum at some point on the boundary of D. **(5)**

b) Check the function u=$\frac{x}{x^2+y^2}$ is harmonic? if yes then find the harmonic conjugate of u. **(5)**

Q5 **a)** Find the Laurent series of f(z)=$\frac{2z-3i}{z^2-3iz-2}$ in the region 1<$|$z$|$ < 2. **(5)**

b) Find the residues of f(z)=$\frac{z^2-2z}{(z+1)^2(z^2+4)}$ **(5)**

Q6 **a)** Find the radius of convergence of $\sum_{n=0}^{\infty} \frac{(3n)!}{2^n(n!)^3} z^n$. **(5)**

b) Evaluate $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)} dx$ **(5)**

Q7 **a)** Let f be meromorphic in a domain D $\subseteq \mathbb{C}$ and have only finitely many zeros and pole in D. if C is a simple closed contour in D such that no zeros or poles of f lies on C ,then $\frac{1}{2\pi i}\oint \frac{f'(z)}{f(z)} dz$ =N-P, where N is no of zeros and P is no of poles of f inside C, each counted according to their order. **(5)**

b) Evaluate:$\int \frac{2z^3-3}{z(z-1+i)^2}$dz ,c consists of $|z|$=2 counter clockwise and $|z|$=1 clockwise. **(5)**

Q8 **a)** Prove that $\left|\oint f(z)dz\right| \leq$ Ml, where M= $\max_{z\in c} |f(z)|$ and l is the length of c. **(5)**

b) Evaluate:$\oint \frac{e^z+z}{z^3-z} dz$,c:$|z|$=$\frac{\pi}{2}$ using Cauchy Residue theorem **(5)**