Registration No : | | | | | | | | | | |

Total Number of Pages: 02 **M.Sc.**

15MMCC204

2nd Semester Back Examination 2017-18

LINEAR ALGEBRA

BRANCH : M.Sc.(MC)

Time : 3 Hours

Max Marks : 70

Q.CODE : C926

Answer Question No.1 which is compulsory and any five from the rest.

The figures in the right hand margin indicate marks.

Answer all parts of a question at a place.

Q1 **Answer the following questions :** **(2 x 10)**

a) If A is invertible what is the inverse of A^T.

b) Describe the smallest subspace of the 2×2matrix space M that contains $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \& \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$.

c) Define null space of a matrix.

d) What do you mean by Hermitian & skew-Hermitian matrix? Give an example.

e) Find the algebraic multiplicity of the given matrix $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

f) The determinant of a unitary matrix has absolute value 1.

g) Prove that for a square matrix A $(A^{-1})^T = (A^T)^{-1}$.

h) If A is invertible and $AB = AC$, Prove that $B = C$

i) Define positive semidefinite.

j) Define minimax maximin.

Q2 **a)** Solve the following system of linear equation by back- substitution for $z, y \& x$. **(5)**

$$\begin{aligned} 2x + 3y + z &= 8 \\ 4x + 7y + 5z &= 20 \\ -2y + 2z &= 0 \end{aligned}$$

b) Find the inverse of the given matrix by using Gauss- Jordan method **(5)**

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 4 & -6 & 0 \\ -2 & 7 & 2 \end{bmatrix}.$$

Q3 **a)** Solve the following system of linear equations by Gauss elimination method **(5)**

$$\begin{aligned} u + v + w &= 0 \\ u + 2v + 3w &= 0 \\ 3u + 5v + 7w &= 1 \end{aligned}$$

b) Show that v_1, v_2, v_3 are linearly independent but v_1, v_2, v_3, v_4 are linearly dependent: $v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $v_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ $v_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $v_4 = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$. **(5)**

Q4 State and prove Fundamental theorem of orthogonality. **(10)**

Q5 a) Find the dimension and a basis for the four fundamental subspaces for $A = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix}$ & $U = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. **(5)**

b) Prove that T^2 is a linear transformation if T is linear(from R^3 to R^3). **(5)**

Q6 a) Find the Eigen values and Eigen vectors of the following matrix $A = \begin{bmatrix} 0 & -i & 0 \\ i & 1 & i \\ 0 & -i & 0 \end{bmatrix}$.what property do you expect for the eigenvectors,and is it true ? **(5)**

b) Find a basis of Eigen vectors that form a unitary system $A = \begin{bmatrix} 0 & 3i \\ -3i & 0 \end{bmatrix}$. **(5)**

Q7 a) Find e^{At} using the given matrix $A = \begin{bmatrix} 1 & 3 \\ 0 & 0 \end{bmatrix}$. **(5)**

b) Find out what type of conic section the following Quadratic form represents and transform it to principal axes **(5)**

$$Q = 17x_1^2 - 30x_1x_2 + 17x_2^2 = 128.$$

Q8 a) Using Gram-Schmidt process find the orthonormal basis for the independent vectors are $a = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $b = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$, $c = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. **(5)**

b) Find the rank of the matrix $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ **(5)**