Fourth Semester Examination – 2007

QUANTITATIVE TECHNIQUES - I

Full Marks – 70

Time – 3 Hours

Answer Question No. 1 which is compulsory and any five from the rest.

The figures in the right-hand margin indicate marks.

1. Answer the following questions : 2×10

   (a) Define the following terms :

   Basic solution, Surplus variable, Pseudo-optimal solution.

P.T.O.

(b) Give a mathematical formulation of the traveling salesman problem.

(c) Write Little's formula.

(d) Using North-West corner method, find the starting basic solution of the transportation problem.

					Supply
	10	2	20	11	15
	12	7	9	20	25
	4	14	16	18	10
Demand	5	15	15	15	

(e) What do you mean by queue-discipline ?

(f) Two red and two black cards are lying face down on a table. If you guess their colours, what is the probability that your guess is right ?

(g) A person has 0.8 probability of passing a test. If the person takes six tests, what

is the probability of his (her) passing at least four test ?

(h) You purchase a lottery ticket worth 20 rupees. If the probability of your winning the first prize of one million rupees is one in one million, what is your expected profit ? [Assume that there is just one prize in this lottery]

(i) A fair coin is tossed 100 times. If X is the number of heads obtained, find the expected value and variance of X.

(j) A book has 2 misprints per page on average. If you open the book at random, what is the probability that there are more than 2 misprints on that page ?

2. (a) Solve graphically :

Maximize $z = 5x + 4y$

subject to : $6x + 4y \leq 24$

$x + 2y \leq 6$

$-x + y \leq 1$

$y \leq 2$

$x, \quad y \geq 0$

(b) Solve :

Minimize $z = 4x + y$

subject to : $3x + y = 3$

$4x + 3y \geq 6$

$x + 2y \leq 4$

$x, \quad y \geq 0$

3. (a) Find the starting solution of the transportation problem given below by Vogel's approximation method :

					Supply
	10	2	20	11	15
	12	7	9	20	25
	14	14	16	18	10
Demand	5	15	15	15	

(b) Babies are born in a sparsely populated state at the rate of one birth every 12 minutes. The time between births follows an exponential distribution.

Find – 5

(i) The average number of births per year

(ii) The probability that no births will occur in any one day.

4. Use branch and bound method to solve the following : 10

Maximize $z = 5x + 7y$

subject to : $2x + y \leq 13$

$5x + 9y \leq 41$

$x, \quad y \geq 0$ are integers.

5. (a) Write short notes on : 5

Finite population models.

(*) The coefficients of a linear equation $ax + b = 0$ are determind by throwing a die twice. Find the probability that the equation has an integral root. 5

6. (a) A bag contains 100 tokens, numbered 1 through 100. If you pick up one token at random, what is the probability that the number on the token is divisible by 2, 3 or 5 but not by all these three numbers ? 5

(b) The mean and the variance of the mathematics marks of an examination are 45 and 15 respectively. Assuming that the marks are normally distributed : 5

(i) find the probability that a randomly picked person has scored more than 70

(ii) find the probability that a randomly picked person has scored less than 30.

7. (a) Define the moment generating function of a random variable X. Find the moment generating function of X if – 5

$$P_r(X=r) = \frac{e^{-\lambda}\lambda^r}{r!}$$

(b) Let $x_1, x_2, \ldots x_n$ be independent random variables with same mean μ and same variance σ^2, find the mean and variance of – 5

$$Y = \frac{x_1 + x_2 + \ldots + x_n - n\mu}{\sqrt{n}\,\sigma}$$

8. (a) The joint p.d.f. of two random variables X and Y, is given by

$$f(x,y) = \begin{cases} 6e^{-2x-3y}, & x>0,\ y>0 \\ 0, & \text{elsewhere;} \end{cases}$$

Find the following probabilities. 5

(i) $P_r(1 \leq X \leq 2, 2 \leq Y \leq 3)$

(ii) $P_r(X \geq 2, Y \geq 2)$

(b) Determine the marginal densities. Are X and Y independent ? 5