Registration No : | | | | | | | | | | |

Total Number of Pages : 02 **B.Tech**

PME3D001

3rd Semester Regular/Back Examination 2017-18

APPLIED MATHEMATICS

BRANCH : MECH

Time : 3 Hours

Max Marks : 100

Q.CODE : B1210

Answer Question No.1 and 2 which are compulsory and any four from the rest.

The figures in the right hand margin indicate marks.

Q1 **Answer the following questions: *multiple type or dash fill up type :*** **(2 x 10)**

a) The function f(z) = $\overline{z}$ at z = 0 is
(a) Analytic (b) not differentiable (c) continuous (d) none.

b) The residue of f(z) = $\frac{\sin z}{z^6}$ $at\ z = 0$ is
(a) $\frac{1}{11}$ (b) $\frac{1}{12}$ (c) $\frac{1}{120}$ (d) none

c) The nature of singularities of the function f(z) = $\frac{z}{1+z^4}$ are
(a) Removal singularity (b) simple poles (c) essential singularity (d) none

d) The Radius of convergence of $\sum_1^\infty \frac{n-1}{(3n+1)!} z^n$ is __________

e) The partial differential equation $yu_{xx} + 2xyu_{xy} + xu_{yy} = 0$ is hyperbolic in
(a) $xy \neq 1$ (b) $xy \neq 0$ (c) $xy > 1$ (d) $xy > 0$

f) The complete integral of the partial differential equation zpq = p + q $p = \frac{\delta z}{\delta x}, q = \frac{\delta z}{\delta y}$ is _______________

g) The complementary solution of $(\mathrm{D} + D' - 1)\ \mathrm{z} = 0$ IS_______________
Where D= $\frac{\delta}{\delta x}$, $D' = \frac{\delta}{\delta y}$

h) If two dice are rolled once then probability of the surface whose sum is at least eight is ___________________

i) If E and F be any two events with P(E∪ $F) = 0.8$, P(E) = 0.4 and
P(E\F) = 0.3 then P(F) is
(a) $\frac{3}{7}$ (b) $\frac{4}{7}$ (c) $\frac{3}{5}$ (d) $\frac{2}{5}$

j) E(E(2^{2017})) is equal to _______________ where E is called Expectation?

Q2 **Answer the following questions: *Short answer type :*** **(2 x 10)**

a) Let f(z) has pole of order m and g(z) has pole of order n , then what is the pole of the f(z)×g(z) .

b) Find the partial differential equations by eliminating arbitrary function of $z = xy + f(x^2 + y^2)$.

c) . Write only the complete integral of the partial differential equations $pqz = p^2qx + q^2py - \sin pq$;$p = \frac{\delta z}{\delta x}, q = \frac{\delta z}{\delta y}$

d) Find the characteristics curve for $y^2u_{xx} - x^2u_{yy} = 0\ with\ x, y \neq 0$.

e) Consider the wave equation $u_{tt} = u_{xx}$,u(x,0)=$\sin x$ $u_t(x, 0) = 1$ then find theme value of u($\pi, \frac{\pi}{2}$).

f) Find the value of $\int_\gamma \frac{(2z^3+5)\,dz}{(z-1)^3}$; $\gamma: |z| = 2$.

g) Find the order of the pole of f(z) = $\frac{z+\sin z}{z^5}$ $at\ z = 0$.

h) If X and Y have the joint density function

$$f(x, y) = \begin{cases} \frac{3}{4} + xy, & 0 < x < 1, 0 < y < 1 \\ 0 & , else \end{cases}$$

find f(y|x) .

i) A Random variable X has density function

$$f(x) = \begin{cases} e^{-x}, & x \geq 0 \\ 0 & , else \end{cases}$$

Then find E(X).

j) Define probability density function for one dimensional continuous Random variable.

Q3 a) Let X and Y be continuous random variable having joint Density Function given by **(10)**

$$f(x,y) = \begin{cases} c(x^2 + y^2), & 0 \leq x \leq 1, \ 0 \leq y \leq 1 \\ 0, & otherwise \end{cases}$$ then find

Find (a) the value of c

(b) P $(\frac{1}{4} < X < \frac{3}{4})$

(c) Expectation of X

(d) Expectation of X+Y?

b) The random variable X takes the values 'n' with probability $\frac{1}{2^n}, n = 1,2,..$ find the moment generating function , mean and variance **(5)**

Q4 a) If X is normally distributed with mean 12 and standard deviation 4 then find **(10)**

(a) P($X \geq 20$)

(b) P($0 \leq X \leq 12$)

b) A random sample of 16 values from a normal population showed a mean of 41.5 inches and the sum of squares of deviations from this mean is equal to 135 square inches. show that assumption of a mean of 43.5 inches for the population is not reasonable. **(5)**

Q5 a) Prove that $\int_0^\infty \frac{\cos ax \ dx}{1+x^2} = \frac{\pi}{2e^a}$ *where a is a real constant* . **(10)**

b) Find the Taylor's series expansion of f(z) = $\frac{z-1}{z+1}$ about z = 0 and z = 1 **(5)**

Q6 a) Evaluate (a) $\int_\gamma \frac{e^z}{z^2(z+1)} dz \ \gamma: |z+1| = 2$ is the positively oriented circle. Using residue theorem. **(10)**

(b) Residue [f(z) = $\frac{2-z}{z^2-z}$] at z = ∞

b) Find the value of the integral $\int_\gamma \frac{1}{(z-3)(z^5-1)} dz$; $\gamma: |z| = 2$ is the positively oriented circle. **(5)**

Q7 a) Evaluate the real integral $\int_0^{2\pi} \frac{(1+2cos\theta)d\theta}{5+4\cos\theta}$ **(10)**

b) Prove that $\int_0^\infty \frac{dx}{1+x^2} = \frac{\pi}{2}$ **(5)**

Q8 a) Solve the wave equation $u_{tt} = u_{xx}$ with u(x,0) = $sin^3 \frac{\pi x}{2}$,0<x<2 $u_t(x,0) = 0$ with u(0,t) = u(2,t)=0 . **(10)**

b) Solve the partial differential equations **(5)**

$(y + zx)$p$-(x + yz)$q= $x^2 - y^2$ *where* $p = \frac{\delta z}{\delta x}, q = \frac{\delta z}{\delta y}$

Q9 a) Find a complete integral of pxy + pq + qy = yz **(10)**

b) Solve $\left(D^2 + D'^2 - 2DD'\right) z = e^{x+2y}$; D= $\frac{\delta}{\delta x}$, $D' = \frac{\delta}{\delta y}$ **(5)**