Registration No : | | | | | | | | | | |

Total Number of Pages : 02

B.Tech
RMA2A001

2nd Semester Regular/Back Examination 2018-19
MATHEMATICS-II
BRANCH : AEIE, AERO, AG, AUTO, BIOMED, BIOTECH, CHEM, CIVIL, CSE, ECE, EEE, EIE, ELECTRICAL, ENV, ETC, IT, MANUTECH, MECH, METTA, METTAMIN, MINERAL, MINING, MME, PE, PLASTIC, PT
Max Marks : 100
Time : 3 Hours
Q.CODE : F131

Answer Question No.1 (Part-1) which is compulsory, any EIGHT from Part-II and any TWO from Part-III.
The figures in the right hand margin indicate marks.

Part- I

Q1 Only Short Answer Type Questions (Answer All-10) **(2 x 10)**

a) Determine value of x for which the matrix $A = \begin{bmatrix} 8 & x & 0 \\ 4 & 0 & 2 \\ 12 & 6 & 0 \end{bmatrix}$ is singular?

b) If a non-homogeneous system of n equations with n unknowns has unique solution, then what is the rank of coefficient matrix?

c) Determine Eigen values of the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 2 & 0 \\ 2 & 3 & 3 \end{bmatrix}$.

d) Define Hermitian matrix and give an example of it.

e) If $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, then determine $div(\vec{r})$.

f) State whether the vector $\vec{v} = yz\hat{\imath} + zx\hat{\jmath} + xy\hat{k}$ is irrotational or not.

g) Derive directional derivative of $f = xyz$ at $P:(-1,1,3)$ in the direction $\vec{a} = \hat{\imath} - 2\hat{\jmath} + 2\hat{k}$.

h) State Gauss Divergence theorem.

i) Determine values of a_0 and a_n in the Fourier expansion of $f(x) = sinx, -\pi < x < \pi$.

j) The function $f(x) = x + cosx$ is even function or odd function or neither even nor odd. Justify your answer.

Part- II

Q2 Only Focused-Short Answer Type Questions- (Answer Any Eight out of Twelve) **(6 x 8)**

a) Calculate rank of the given matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{bmatrix}$.

b) Explain that product of two unitary matrices is unitary.

c) Solve the system of equations $x + y - z = 9, 8y + 6z = -6, -2x + 4y - 6z = 40$ by using Gauss Elimination method.

d) Calculate inverse of matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$ by Gauss Jordan method.

e) For any scalar function f(x,y,z) and vector function $\vec{v} = [v_1, v_2, v_3]$, Prove that
   i. $Curl(grad\, f) = 0$
   ii. $Div(curl\, \vec{v}) = 0$

f) Calculate length of the curve $\vec{r}(t) = acost\,\hat{\imath} + asint\,\hat{\jmath} + 4t\,\hat{k}$ from $(a,0,0)$ to $(a,0,8\pi)$.

g) Formulate Fourier Cosine transform and Fourier Sine transformation of

$$f(x) = \begin{cases} 1, 0 \le x < 1 \\ 0, \quad x \ge 1 \end{cases}$$

h) Develop Half range Fourier Cosine series of the function $f(x) = x^2, 0 < x < 2$.

i) Evaluate the integral
$\int_C (y^2 dx - x^2 dy)$, C: Straight line segment from $(0,0)$ to $(1,1)$.

j) Design Fourier series of $f(x) = |x|, -2 < x < 2, p = 4$.

k) Explain that the given line integral

$$\int_{(0,2,3)}^{(1,1,1)} yz \sinh xz\, dx + \cosh xz\, dy + xy \sinh xz\, dz$$

is independent of path and hence find the value of integral.

l) Calculate unit normal vector of the surface $r(u,v) = [u \cos v, u \sin v, u^2]$.

Part-III

Only Long Answer Type Questions (Answer Any Two out of Four)

Q3 Diagonalize $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$. **(16)**

Q4 Evaluate eigen values and eigen vectors for the given matrix, **(16)**

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}.$$

Q5 Verify Stokes's theorem for $F = [x, y, z]$ and surface S the paraboloid $z = f(x,y) = 1 - (x^2 + y^2), z \geq 0$. **(16)**

Q6 a) Using Fourier integral representation, Prove that **(10)**

$$\int_0^\infty \frac{\cos x\omega + \omega \sin x\omega}{1 + \omega^2} d\omega = \begin{cases} 0, & if\ x < 0 \\ \frac{\pi}{2}, & if\ x = 0 \\ \pi e^{-x}, & if\ x > 0 \end{cases}$$

b) Evaluate Fourier series of $f(x) = x^2, -\pi < x < \pi$. **(6)**