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Total Number of Pages : 02 **M.Sc.I**
FMCE903

9th Semester Regular Examination 2019-20
ALGEBRAIC GRAPH THEORY
BRANCH : M.Sc.I(MC)
Time : 3 Hours
Max Marks : 70
Q.CODE : HR208
Answer Question No.1 which is compulsory and any FIVE from the rest.
The figures in the right hand margin indicate marks.

Q1 **Answer the following questions :** **(2 x 10)**

a) Define the spectrum of a graph and find it for K_4.

b) What are the values of the rank and co-rank of a graph Γ with 'n' vertices, 'm' edges and 'c' components?

c) What are the components of an elementary graph?

d) If *g(Γ)* denotes the number of edges in a smallest cycle (girth) of a graph *Γ*, then for n≥3, and a≥2, what are the values of $g(K_n)$ and $g(K_{a,a})$?

e) Define the tree number of a graph Γ. What is the tree number of K_n?

f) Is the line graph L(Γ) of a regular graph Γ of degree 'k' a regular graph? If yes then what is the degree of L(Γ)?

g) What is the chromatic polynomial of a tree T containing 'n' vertices and what is it'schromatic number?

h) Define the cone and the suspension of a graph Γ.

i) Define the Tutte polynomial.

j) Define Cayley graph.

Q2 Prove that the coefficients of the characteristic polynomial $\chi(\Gamma;\lambda) = \lambda^n + c_1\lambda^{n-1} + c_2\lambda^{n-2} + \cdots + c_n$ of a graph Γ satisfy : **(10)**
(i) $c_1 = 0$.
(ii) $-c_2$ is the number of edges of Γ ; and
(iii) $-c_3$ is twice the number of triangles in Γ.

Q3 If Γ is a regular graph of degree *k* with *n* vertices and *m* edges, then prove that$\chi(L(\Gamma);\lambda) = (\lambda + 2)^{m-n}\chi(\Gamma;\lambda + 2 - k)$. **(10)**

Q4 **a)** Prove that any square submatrix of the incidence matrix D of a graph Γ has determinant equal to 0 or +1 or -1. **(5)**

b) Find the tree number of the line graph L(Γ) of a k-regular graph Γ. **(5)**

Q5 Let $0 \le \mu_1 \le \mu_2 \le \ldots \le \mu_{n-1}$ be the Laplacian spectrum of a graph Γ with n vertices. Then prove that $\kappa(\Gamma) = \frac{\mu_1\mu_2\cdots\cdots\mu_{n-1}}{n}$. Again prove that If Γ is a connected and k-regular graph and its spectrum is Spec Γ = $\begin{pmatrix} k & \lambda_1 \ldots\ldots & \lambda_{s-1} \\ 1 & m_1 \ldots\ldots & m_{s-1} \end{pmatrix}$, then $\kappa(\Gamma) = n^{-1}\chi'\Gamma(k)$,where$\chi'$ denote the derivative of the characteristic polynomial χ . **(10)**

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Q6 **a)** Prove that the chromatic polynomial satisfies the relation $C(\Gamma;u) = C\left(\Gamma^{(e)};u\right) - C(\Gamma_{(e)};u)$, where $\Gamma^{(e)}$& $\Gamma_{(e)}$ denotes the graph obtained from the graph Γ by deleting and contracting the edge 'e' respectively. **(5)**

b) Prove that a graph is bipartite iff it is bi-chromatic. **(5)**

Q7 **a)** If $A \subseteq B \subseteq A^{\lambda}$,then prove that $B^{\lambda} = A^{\lambda}$, where A & B are two subgraphs of a graph Γ. **(5)**

b) If $A \subseteq B\ and\ r(B) \neq r_0$, then prove that $\lambda(B) \in A^{\lambda}$. **(5)**

Q8 **a)** If a connected graph is edge-transitive but not vertex-transitive, then prove that it is bipartite. **(5)**

b) Let λ be a simple eigenvalue of a graph Γ and **x** be the corresponding eigenvector with real components. If the permutation matrix **P** represents an automorphisim of Γ, then **Px**= ±**x.** **(5)**