Registration No :

Total Number of Pages : 02

M.Sc.I
FMCC602

6th Semester Regular / Back Examination 2017-18
COMPLEX ANALYSIS
BRANCH : M.Sc.I(MC)
Time : 3 Hours
Max Marks : 70
Q.CODE : C235

Answer Question No.1 which is compulsory and any five from the rest.
The figures in the right hand margin indicate marks.

Q1. **Answer the following questions :** **(2 x 10)**

a) Find all cube roots of $\sqrt{2}+i\sqrt{2}$.

b) Find the principal value of $(1+\sqrt{3}i)$ and its polar form.

c) Find the radius of convergence of the power series $\sum_0^\infty \frac{z^n}{2^n+1}$.

d) Use Cauchy's integral formula to evaluate $\frac{1}{2\pi i}\oint_C \frac{z^2+5}{z-3}dz$ where C is $|z| = 4$.

e) Find a bilinear Transformation which maps the points $-1, 1, \infty$ onto $-i, -1, i$ respectively.

f) Find the fixed point of the transformation $w = \frac{(2+i)z-2}{z+i}$.

g) Prove that the function $f(z) = \bar{z}$ is nowhere differentiable.

h) Is $u = x^3 - 3xy^2$.harmonic? If yes, then find its harmonic conjugate and the corresponding analytic function in terms of z.

i) Determine the residue of $\frac{z^2}{z^2+a^2}$ at its poles.

j) What is essential singularity of an analytic function $f(z)$ in a domain? Explain with example.

Q2. a) Show a Mobius transformation has ∞ as its only fixed point if and only if it is a translation. **(5)**

b) Prove that cross ratio remains invariant under a bilinear transformation. **(5)**

Q3. a) Use derivatives of analytic function to evaluate the integral $\int_\gamma \frac{\sin z}{z^3}dz$; $\gamma(t) = e^{it}, 0 \le t \le 2\pi$. **(5)**

b) If γ is a piecewise smooth curve and $f: [a, b] \to C$ is continuous then prove that $\int_a^b f\, d\gamma = \int_a^b f(t)\gamma'(t)dt$. **(5)**

Q4. a) For a doubly connected domain D with outer boundary curve C_1 and inner C_2 such that $f(z)$ is analytic in any domain D_1 which contains D and its boundary curve then show that $\oint_{C_1} f(z)dz = \oint_{C_2} f(z)dz$. **(5)**

b) Let $f(z)$ be analytic in a simply connected domain D. Then for any point z_0 in D and any simply closed path γ from a to b in D that encloses z_0 show that $f^{(n)}(z_0) = \frac{n!}{2\pi i}\oint_\gamma \frac{f(z)}{(z-z_0)^{n+1}}dz.$ **(5)**

Q5. a) If $f(z)$ is analytic on and within a circle C given by $|z - z_0| = R$ and if $|f(z)| \leq M$ for every $z\ in\ C$ then prove that $\left|f^{(n)}(z_0)\right| \leq \frac{n!\,M}{R^n}$. **(5)**

b) If $f(z)$ is a bounded entire function in the whole complex plane , then show that $f(z)$ must be constant. **(5)**

Q6. a) State and prove Moreras theorem. **(5)**

b) If γ is a positively oriented simple closed contour in G and f is analytic in the region G except at a finite number of points $z_1, z_2, \ldots, z_n$ inside γ then prove that $\int_\gamma f(z)dz = 2\pi i \sum_{k=1}^{n} Res(f; z_k)$. **(5)**

Q7. a) Prove that the Laurent's expansion is unique where $f(z)$ is analytic in the annulus between two concentric circles C_1 and C_2 with centre at $z = a$ and radii R_1 and R_2 respectively. **(6)**

b) Find the Laurent's series for $f(z) = \frac{4z+4}{z(z-3)(z+2)}$ valid in $|z| > 3$. **(4)**

Q8. Use the method of Contour integration to evaluate the following integrals **(answer any TWO) :** **(5 x 2)**

a) $\int_0^\infty \frac{dx}{(1+x^2)^2}$

b) $\int_0^\infty \frac{sinmx}{a^2+x^2}dx$

c) $\int_0^\pi \frac{1+2cos\theta}{5+4cos\theta}d\theta$