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M.Sc.I FMCE407

4<sup>th</sup> Semester Back Examination 2018-19 MATH-IV

BRANCH: M.Sc.I(AP)
Time: 3 Hours
Max Marks: 70

Q.CODE: F366

Answer Question No.1 which is compulsory and any five from the rest.

The figures in the right hand margin indicate marks.

## Q1 Answer the following questions:

 $(2 \times 10)$ 

- Determine the smaller root of the equation  $x^2 400x + 1 = 0$  by using four digits arithmetic.
- b) Obtain an interval which contains a root of the equation  $f(x) = \cos x xe^x = 0$ .
- c) What is the convergence criterion for Gauss Jacobi iteration method?
- d) Perform two iterations of the Newton-Raphson method to find the smallest positive root of the equation  $f(x) = x^3 5x + 1 = 0$ .
- e) Write the pitfalls of Newton Raphson's method.
- f) Define interpolation and extrapolation.
- g) What is Least Square Approximation?
- h) Write the formula for Euler's Modified Method.
- i) Find the approximate value of  $I = \int_{0}^{1} \frac{dx}{1+x}$  by using Simpson's  $\frac{1}{3}$  rule.
- j) Compare composite Simpson's  $\frac{1}{3}$  rule and Simpson's  $\frac{3}{8}$  rule.
- Q2 a) Perform four iterations of the Newton-Raphson method to obtain the approximate value of  $(17)^3$  starting with the initial approximation  $x_0 = 2$ .
  - b) Use Regular-Falsi methods to determine the root of the equation (5)  $\cos x xe^x = 0$  correct up to 3 decimal places.
- Q3 a) Given that f(0) = 1, f(1) = 3, f(3) = 55, find the unique polynomial of degree 2 or less, that fits the given data. Also, find the bound on error.
  - b) Construct Newton's forward interpolating polynomial for the following data: (5)

| Х | 4 | 6 | 8 | 10 |  |
|---|---|---|---|----|--|
| У | 1 | 3 | 8 | 16 |  |

Also, evaluate y for x = 5.

- Q4 a) State and prove the existence and uniqueness theorem of Lagrange (5) interpolating polynomial.
  - b) Evaluate  $\int_0^1 \frac{dx}{1+x}$  by using Gauss- Legendre 3-point rule. Also estimate the error. (5)

- **Q5** a) Use Simpson's  $\frac{1}{3}$  rule to approximate the value of  $\tan^{-1} 4$  by evaluating  $\int_0^4 \frac{dx}{1+x^2}$ . (5)
  - b) Solve: y' = x + y, y(0) = 1 by using Euler's method. (5) Also, find the value of y at x = 0.1.
- Q6 Discuss the condition for existence of inverse of a matrix and find the inverse (10)

of the matrix 
$$\begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix}$$

by using LU decomposition method. Take  $u_{11} = u_{22} = u_{33} = 1$ .

- Q7 Solve the system of equations  $\begin{bmatrix} 4 & 1 & 1 \\ 1 & 5 & 2 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -6 \\ -4 \end{bmatrix}$  using the Gauss Jacobi (10) and Gauss-Seidel iteration method. Take the initial approximation as
- $\begin{bmatrix} 0.5, & -0.5, & -0.5 \end{bmatrix}$ . Also, find the exact solution. Q8 Write short answer on any TWO: (5 x 2)
  - a) LUDecomposition
  - b) Uniform Approximation
  - c) Gauss Legendre Integration Method