Registration No: | | | | | | | | | | |

Total Number of Pages : 02

M.Sc.I
FMCC303

3rd Semester Regular Examination 2017-18
CALCULUS I & ANALYTICAL GEOMETRY
BRANCH : M.Sc.I(MC)
Time : 3 Hours
Max Marks : 70
Q.Code : B856

Answer Question No.1 which is compulsory and any Five from the rest.
The Right hand margin indicates marks.

Q1 **Answer the following questions:** **(2x10)**

a) Define asymptote to a curve. How many asymptotes are obtained for a 6th degree equation.

b) Find radius of curvature of a curve $r = a\cos(2\theta)$.

c) Define multiple points. What is the necessary and sufficient condition for any point (x, y) on $f(x, y) = 0$ to be a multiple point?

d) In what condition a Cartesian equation is symmetric about axis.

e) Find the center and radius of the sphere $x^2 + y^2 + z^2 - 3x + 2y + z - 12 = 0$.

f) Find the condition for the plane $ax + by + cz + d' = 0$ to be the tangent to the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$.

g) Define right circular cone.

h) Find the cone whose vertex is at origin and whose guiding curve is the intersection of the surface $2x^2 + 3y^2 + 4z^2 = 5$ and the plane $x + y + z = 2$.

i) Find the equation of the sphere whose end point of diameter is $(1, -2, 3), (3, 0, 1)$.

j) Find the tangent at the origin $x^2(x^2 + y^2) = a(x - y)$.

Q2 a) Show that the curvature of the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ on the Folium $x^3 + y^3 = 3axy$ is $\frac{-8\sqrt{2}}{3a}$ **(5)**

b) Find the radius of curvature at the origin of the curve $x^4 - y^4 + x^3 - y^3 + x^2 - y^2 + y = 0$. **(5)**

Q3 a) Find the asymptotes of the curves $xy^2 - x^2y - 3x^2 - 2xy + y^2 + x - 2y + 1 = 0$. **(5)**

b) Find the asymptotes of the curve $x^2y - xy^2 + xy + y^2 + x - y = 0$, and show that they cut the curve again in three points which lie on the line $x + y = 0$. **(5)**

Q4 a) Find the position and nature of the multiple points of the curve $x^4 + 4ax^3 + 4a^2x^2 - b^2y^2 - 2b^3y - a^4 - b^4 = 0$. **(5)**

b) Find the asymptotes of the curve $r\theta = a$. **(5)**

Q5 **Trace the following curves.**

a) $r = a\sin(3\theta)$. **(5)**

b) $y^2(a^2+x^2)=x^2(a^2-x^2)$. **(5)**

Q6 a) Find the values of 'a' such that the plane $x+y+z=a\sqrt{3}$ will touch the sphere $x^2+y^2+z^2-2x-2y-2z-6=0$. **(5)**

b) Find the equation of the sphere through the circle $x^2+y^2+z^2-4=0=x^2+y^2+z^2-2x-2y-4$, whose center lies on the plane **(5)**

$$x+y+z=1.$$

Q7 a) Find the equation of the cone whose vertex is $(1,1,0)$ and whose guiding curve is the circle $x^2+z^2=4, y=0$. **(5)**

b) Prove that the equation $ax^2+by^2+cz^2+2ux+2vy+2wz+d=0$ represents a cone if $\frac{u^2}{a}+\frac{v^2}{b}+\frac{w^2}{c}-d=0$. **(5)**

Q8 a) Find the equation of the cylinder whose generators are parallel to the line $\frac{x}{1}=\frac{y}{1}=\frac{z}{1}$ and whose guiding curve is the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1, z=1$. **(5)**

b) Find the equation of the right circular cone with vertex $(1,-2,-1)$, semivertical angle $\frac{\pi}{3}$ and axis $\frac{x-1}{3}=\frac{y+2}{-4}=\frac{z+1}{5}$. **(5)**