**Registration No :**

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**Total Number of Pages : 02** **M.Sc.I**

**FMCE207**

**2nd Semester Back Examination 2017-18**
**MATHEMATICS-II**
**BRANCH : M.Sc.I(AC)**
**Time : 3 Hours**
**Max Marks : 70**
**Q. CODE : C931**
**Answer Question No.1 which is compulsory and any five from the rest.**
**The figures in the right hand margin indicate marks.**
**Answer all parts of a question at a place.**

**Q1** **Answer the following questions :** **(2 x 10)**

a) Define limit point of a set. Find the derived set of the set $\{x : 0 \le x < 1\}$.

b) What is an open set? Give an example of an open set which is not an interval.

c) Show that, if a set is closed, its complement is open.

d) Find the limit inferior and superior of the sequence $\{(-1)^n n, n \in N\}$.

e) What is the necessary and sufficient condition for the convergence of a sequence?

f) Show that a set of natural numbers is not a group with respect to addition.

g) Show that N is a normal subgroup of G iff $gNg^{-1}=N$.

h) Define homomorphism from a group $G$ to another group $\bar{G}$ .What is kernel of Homomorphism.

i) Define ring with unity element and commutative ring.

j) If $R$ is a ring, then for all $a,b \in R$, then show that $(-a)(-b) = ab$.

**Q2** a) Show that the real number field is Archimedean. **(5)**

b) Show that the intersection of any finite number of open sets is open. **(5)**

**Q3** a) Show that the union of two closed sets is a closed set. **(5)**

b) Show that a sequence cannot converge to more than one limit. **(5)**

**Q4** a) State and Prove Bolzano Wierstrass theorem for sequences. **(5)**

b) Show that $\lim_{n\to\infty} \sqrt[n]{n} = 1$. **(5)**

**Q5** a) Explain Cauchy's root test for convergence of an infinite series. **(5)**

b) Test for the convergence of the series $\sum \frac{1}{n^{1+\frac{1}{n}}}$. **(5)**

**Q6** **a)** Let $a, b \in G$, where $G$ is a group. If $H$ is a subgroup of $G$, then show that the relation $a \equiv b$ mod $H$ is an equivalence relation. **(5)**

**b)** Let $H$ and $K$ be finite subgroups of a group $G$, then show that **(5)**

$$O(HK) = \frac{O(H)O(K)}{O(H \cap K)}.$$

**Q7** **a)** Show that the intersection of any two normal subgroups of a group is a normal subgroup. **(5)**

**b)** If $\phi$ is a homomorphism of $G$ into $\bar{G}$, then show that (i) $\phi(e) = \bar{e}$ and (ii) $\phi(x^{-1}) = \phi(x)^{-1}$. **(5)**

**Q8** **a)** Show that, the set of integers under the usual operations of addition and multiplication is a ring. **(5)**

**b)** Prove that the homomorphism $\phi$ from a ring $R$ into a ring $R'$ is an isomorphism Iff the kernel $I(\phi) = 0$. **(5)**