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MSc.I
FMCC602

6th Semester Regular Examination– 2016-17
COMPLEX ANALYSIS
BRANCH(S): Mathematics and Computing
Time: 3 Hour
Max marks: 70
Q Code : Z191

Answer Question No.1 which is compulsory and any five from the rest
The figures in the right hand margin indicate marks

Q1 Answer the following questions: (2 x 10)

a) If $f(z)$ is differentiable at z_0 show that it is continuous at z_0.

b) Determine the radius of convergence of the power series $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^{n^2} z^n$.

c) Find the fixed points of $T(z)=\frac{z+1}{z-1}$.

d) Find all points at which the mapping $w=z^2+\frac{1}{z^2}$ is not conformal.

e) Evaluate $\int_{\gamma} \frac{dz}{z^2}$ where γ is defined by $|z|=d, d>0$.

f) State Morera's theorem.

g) Classify the isolated singular points of $\frac{(1+z)\cos z}{z}$.

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h) State Argument principle.

i) Find the residue at the singular point of $f(z)=\frac{\sin 2z}{z^6}$.

j) Evaluate $\int_C \frac{e^z dz}{\pi z-i}$, where C is $|z|=1$.

Q2 a) Show that an analytic function with constant modulus is constant. [4]

b) Show that $f(z)=e^{-z^{-4}}$, $z\neq 0$ and $f(0)=0$ is not analytic at $z=0$ although Cauchy-Riemann equations are satisfied there. [6]

Q3 a) Define cross ratio and prove that the cross ratio is real if and only if [5]

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four points lie on a circle.

b) Find a Linear Fractional Transformation that maps left half plane into the unit disk. [5]

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Q4 a) State and prove Cauchy's integral formula. [5]

b) Evaluate $\int_C \frac{\cosh(\pi z)}{z(z^2+1)} dz$ counterclockwise around the contour $C: |z| = 2$. [5]

Q5 State and prove Maximum modulus theorem [10]

Q6 a) Represent the function $f(z) = \frac{1}{z(z^2-3z+2)}$ in Laurent series for the regions (i) $0 < |z| < 1$ (ii) $1 < |z| < 2$. [5]

b) Evaluate $\int_0^\infty \frac{\cos x}{x^2+a^2} dx$ [5]

Q7 Using residue theorem evaluate the following integrals [5+5]

(a) $\int_0^{2\pi} \frac{\cos\theta}{3+\sin\theta} d\theta$ (b) $\int_{-\infty}^{\infty} \frac{dx}{x^4+16}$

Q8 a) Prove that if a function $f(z) = u(x,y) + iv(x,y)$ is analytic in a domain D, then u and v are harmonic in D. [2]

b) State and prove the fundamental theorem of algebra. [4]

c) Find the zeros and discuss the nature of singularities of [4]

$$f(z) = \frac{z-2}{z^2} \sin\frac{1}{z-1}$$

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