Registration no: | | | | | | | | | | |

Total Number of Pages: 2 **M.Sc.I**
FMCC402

4^{th} Semester Regular Examination – 2016-17
Geometry of Curves & Surfaces
Branch: M.Sc.I(MC)
Time: 3 Hours
Max Marks: 70
Q.CODE:Z1143

Answer Question No.1 which is compulsory and any five from the rest.
The figures in the right hand margin indicate marks.

Q1 **Answer the following questions:** **(2 x 10)**

a) Define curve and describe different ways of representation.
b) Define Curvature and Torsion.
c) Describe Osculating Plan, Normal Plane and Rectifying plane.
d) Describe Family of surfaces .
e) Define Envelops.
f) Define Involutes and Evolutes.
g) Show that $\vec{r}''' = K'\hat{n} - K^2\hat{t} + K\tau\hat{b}$
h) Define Parameterized Curve..
i) Define Curvilinear Co-ordinates.
j) Describe edge of regression.

Q2 **a)** State and prove Serret & Frenet Formula. **(5)**
b) Find the Osculating plane at a point 't' on the helix **(5)**
x = a cos t, y = a sin t, z = ct

Q3 **a)** Show that curvature and torsion of any curve, $r = r(t)$ are given by **(5)**

$$K = \frac{|\dot{\vec{r}}\ x\ \ddot{\vec{r}}|}{|\dot{\vec{r}}|^3} \quad \& \quad T = \frac{|\dot{\vec{r}}\ \ \ddot{\vec{r}}\ \ \dddot{\vec{r}}|}{|\dot{\vec{r}}\ \ x\ \ddot{\vec{r}}|^2}$$

b) Prove that curvature and torsion of the circular helix are constant. **(5)**

Q4 **a)** A parametrized curve has a unit speed of reparametrization iff it is a regular curve. **(5)**
b) Prove that , at points common to the surface **(5)**
$$a\,(yz + zx + xy) = xyz$$
and a sphere whose center is origin .The tangent plane to the surface makes intercept on the axis whose sum is constant.

Q5 a) Prove that the tangent plane to the surface $xyz = a^3$ And the co-ordinate planes bound a tetrahedron of constant volume. **(5)**

b) Show that the sum of the squares of the intercepts on the coordinate axes made by the tangent plane to the surface $x^{3/2} + y^{3/2} + z^{3/2} = a^{3/2}$ is constant. **(5)**

Q6 a) Find the envelope of the family of planes **(5)**

$$3a^2x - 3ay + z = a^3$$

And show that its edge of regression is the curve of intersection of the surfaces

$$xz = y^2, \quad xy = z$$

b) On the surface of revolution $x = u\cos\emptyset, y = u\sin\emptyset, z = f(u)$ what are the parametric curves u= constant and what are curves ϕ= constant **(5)**

Q7 a) Prove that the surface $xy = (z-c)^2$ is a developable surface. **(5)**

b) Describe the first order magnitudes. **(5)**

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Q8 a) Prove that F=0 is the necessary and sufficient condition that the parametric curves may form an orthogonal system . **(5)**

b) Find the envelope of the plane **(5)**

$$\frac{x}{a+u} + \frac{y}{b+u} + \frac{z}{c+u} = 1$$

Where u is the parameter and determine the Edge of Regression..