Registration no: | | | | | | | | | | |

Total Number of Pages: 2

M.Sc
MMCC301

3rd Semester Regular / Back Examination – 2017-18
Functional Analysis
Branch: M.Sc.(MH)
Time: 3 Hours
Max marks: 70
Q Code:B586
Answer Question No.1 which is compulsory and any five from the rest.
The figures in the right hand margin indicate marks.

Q1 **Answer the following questions:** **(2 x 10)**

a) What is orthonormal sets ?

b) Define resolvent set and spectrum set.

c) $F: X \rightarrow Y$ be linear and continuous for what condition in X and Y the map is uniformly continuous.

d) Define strictly convex, convex and uniformly convex.

e) For what condition in p and q, the dual of c_{00} with the norm $\| \quad \|_p$ is linearly isometric to l_q.

f) Define normal,unitary and self adjoint operator.

g) State schwartz inequality in an inner product space.

h) What is Minkowski space ?

i) Define positive operator.

j) What do you mean by re-presenter of f.

Q2 a) State & prove Acoll's Lemma. **(5)**

b) Let X be a normed space Y be a closed subspace of X and $Y \neq X$ .Let r be a real number such that $0 < r < 1$. Then prove that there exists some $x_r \in X$ such that **(5)**

$$\|x_r\| = 1 \quad \text{and} \quad r < dist(x_r, Y) \leq 1.$$

Q3 State and prove Hahn-Banach extension theorem. **(10)**

Q4	**a)**	State and prove Closed graph theorem.	**(5)**
	b)	Prove that if X is finite dimensional then X is complete.	**(5)**
Q5	**a)**	State & prove Bounded Inverse Theorem.	**(5)**
	b)	Let X and Y be Banach space and $F: X \to Y$ be linear map which is closed and surjective. Then prove that F is continuous and open.	**(5)**
Q6	**a)**	Prove that if a normed dual space X' is separable, then so also normed space X is separable.	**(5)**
	b)	Let X is a Hilbert space and $\sum_n \|k_n\|^2 < \infty$, then prove that $\sum_n k_n u_n$ converges in X.	**(5)**
Q7	**a)**	Let H be a Hilbert space. For $f \in H'$, let y_f be the representer of f in H. Then prove that the map $T: H' \to H$ given by $T(f) = y_f$ is a surjective conjugate linear.	**(5)**
	b)	Let H be a Hilbert space. Let $A, B \in BL(H)$ then prove that $(i)\ (A+B)^* = A^* + B^*$, $(ii)\ (AB)^* = B^*A^*$.	**(5)**
Q8		Let $K = C$ and $A \in BL(H)$ then prove that there are unique self adjoint operators B and C on H such that $A = B + iC$. Again prove that A is normal if and only if BC=CB, A is unitary if and only if BC=CB and $B^2 + C^2 = I$,and A is self adjoint if and only if C=0.	**(10)**