Registration No:

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M.Sc.
MMCC302

3rd Semester Regular/Back Examination 2017-18
Matrix Algebra
BRANCH : M.Sc.(MC)
Time: 3 Hours
Max Marks: 70
Q.CODE: B588
Answer Question No.1 which is compulsory and any five from the rest.
The figures in the right hand margin indicate marks.

Q1 **Answer the following questions :** **(2 x 10)**

a) If $A=\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$ & $X = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$ $B =\begin{bmatrix} 50 \\ 122 \end{bmatrix}$ Then express B as a linear combition of columns of A .

b) If G is a triangular matrix then what is the value of det(G) ?

c) When A is said to be positive definite if A is a nxn real symmetric matrix.

d) What is Cauchy Schwarz Inequelity?

e) For any real number P $\geq$1 define P-norm?

f) If $A=\begin{bmatrix} 1000 & 999 \\ 999 & 998 \end{bmatrix}$ Find $\|A\|_1$, $\|A\|_\infty$, $\|A^{-1}\|_\infty$ $\|A^{-1}\|_1$

g) If $x=(x_1 \; x_2 \; x_3)^T$ & $y=(y_1 \; y_2 \; y_3)^T$. Define inner product of x & y.

h) When a set of vectors q_1, q_2, q_3, ……q_k is said to be orthonormal?

i) Find the relation between R(A) perp. and $N(A^T)$.

j) Determine the eigen values for the following matrix, $A=\begin{pmatrix} 3 & 1 \\ 6 & 2 \end{pmatrix}$.

Q2 **Answer the questions :** **(5)**

a) Solve the following system

$$\begin{bmatrix} 8 & -4 & 0 \\ -4 & 8 & -4 \\ 0 & -4 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

b) Find the matrix equation for the differential equation **(5)**
-u''(x)+cu'(x)+du(x_i) = f(x) 0 < x< 1
With boundary condition
u(0) = 0, u(1) = 0
Using
M = 6 , h = 1/6

Q3 **a)** Use column oriented version of forward substitution solve the following **(5)**
lower Triangular system   (5)
$$\begin{bmatrix} 5 & 0 & 0 \\ 2 & -4 & 0 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 15 \\ -2 \\ 10 \end{bmatrix}$$

b) Let $A = \begin{bmatrix} \begin{pmatrix} 4 & -2 & 4 & 2 \\ -2 & 10 & -2 & -7 \\ 4 & -2 & 8 & 4 \\ 2 & -7 & 4 & 7 \end{pmatrix} \end{bmatrix}$ Use Cholesky's Method to calculate **(5)**

Cholesky factor R.

Q4 **a)** Let M be any n x n non singular matrix and let A = M^{T}M, Then prove that A is positive definite. **(5)**

b) If A = $\begin{pmatrix} 2 & 4 & 2 & 3 \\ -2 & -5 & -3 & -2 \\ 4 & 7 & 6 & 8 \\ 6 & 10 & 1 & 12 \end{pmatrix}$ & b= $\begin{pmatrix} -3 \\ 3 \\ -1 \\ -16 \end{pmatrix}$ **(5)**

Then calculate L & U such that A = LU &solve the system Ax = b, where x = $\begin{pmatrix} x_1 & x_2 & x_3 & x_4 \end{pmatrix}^T$

Q5 **a)** Solve the system $\begin{pmatrix} 2 & 2 & -4 \\ 1 & 1 & 5 \\ 1 & 3 & 6 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 10 \\ -2 \\ -5 \end{pmatrix}$, By Gaussian Elimination with **(5)**

partial pivoting.

b) Prove that $\|A\|_1 = max_{1 \le j \le n} \sum_{i=1}^{n} |a_{ij}|$ **(5)**

Q6 **a)** Use QR – decomposition to solve the system $\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ **(5)**

b) Let$x, y \in R^n$ with $x \ne y$ but $\|x\|_2 = \|y\|_2$. Then prove that there is a unique reflector Q **(5)**
such that Qx = y

Q7 **a)** Let A $\in R^{n \times m}$ & b $\in R^n$, n>m & suppose that 'A' has full rank than the least square problem for the over determined system Ax = b has a unique solution which can be found by solving the non singular system $\check{R}x$ = ĉ, where $\begin{pmatrix} ĉ \\ d \end{pmatrix}$ = c = Q^{T}b , $\check{R} \in R^{m \times m}$ & Q $\in R^{n \times n}$ **(5)**

b) A $\in R^{n \times n}$ can be expressed as product A=QR, using reflectors where Q is a orthogonal matrix and R is a upper triangular matrix **(5)**

Q8 **a)** Find QR – Transform of A = $\begin{pmatrix} 1 & -1 & 4 \\ 1 & 4 & -2 \\ 1 & 4 & 2 \\ 1 & -1 & 0 \end{pmatrix}$ Using Gram – Schmidt Process **(5)**

b) Let S be any subspace of R^n . Then for every x $\in$ R^n , there exist unique element s $\in$ S and s perp. $\in$ S perp. for which x = s + s perp. **(5)**