Registration no:

Total Number of Pages: 01

2nd Semester Regular Examination – 2016-17
PARTIAL DIFFERENTIAL EQUATION
BRANCH(S): M.Sc.(Z1163MH)
Time: 3 Hours
Max Marks: 70
Q.CODE:Z1163
Answer Question No.1 which is compulsory and any five from the rest.
The figures in the right hand margin indicate marks.

Q1   **Answer the following questions:**   (2 ×10)
  a) Find the partial differential equation by eliminating arbitrary function from $f(x + y + z, x^2 + y^2 + z^2) = 0$.
  b) Find the general integral of the linear partial differential equation $y^2 p - xyq = x(z - 2y)$.
  c) Write the Charpit's auxiliary equation..
  d) Classify the partial differential equation $u_{xx} + 4u_{xy} + 4u_{yy} = 0$.
  e) Write the Laplacian in cylindrical and spherical coordinate form.
  f) Solve $4u_x + u_y = 3u, u(0, y) = e^{-5y}$.
  g) Write the two substitutions are introduced in D'Alemberts method to solve Partial differential equations.
  h) Write the Duhamel's Principle..
  i) Write the necessary and sufficient condition that a surface be an integral surface of a partial differential equation.
  j) Show that the partial differential equation $u_{tt} - c^2 u_{xx} = 0$ is hyperbolic and find its Canonical form.

Q2 **a)** Solve the equations $xp = yq, z(xp + yq) = 2xy$ are compatible and solve them.   **(5)**
     b) Solve using Charpit's Method $z^2 = pqxy$.   **(5)**

Q3 **a)** Show that the only solution of $\nabla^2 u = 0$ depending on $r = \sqrt{x^2 + y^2}$ is $u = a \ln r + b$ with   **(5)**
          constants 'a' and 'b'.
     b) Find the electrostatic potential between two concentric spheres of radii $r_1 = 2cm$ and $r_2 = 4cm$ **(5)**
          kept at the potential $U_1 = 220volt$, and $U_2 = 140\ volt$ respectively.

Q4   Show that $u_n = r^n \cos n\theta, u_n = r^n \sin n\theta, n = 0,1,...$ are solution of Laplace's equation   **(10)**
          $\nabla^2 u = 0$ with $\nabla^2 u = u_{rr} + \frac{1}{r}u_r$

Q5 **a)** Find the surface which intersects the surfaces of the system $z(x + y) = c(3z + 1)$ orthogonally   **(5)**
          and which passes through the circle $x^2 + y^2 = 1, z = 1$.
     b) Find the solution of the equation $z = \frac{1}{2}(p^2 + q^2) + (p - x)(q - y)$ which passes through the   **(5)**
          X-axis.

Q6   Solve $u_t - u_{xx} = 0, u(x, 0) = u(0, t) = 0; u(1, t) = t$ for $0 \le x \le 1, 0 \le t \le 0.5$ **(10)**

Q7   Derive the two dimensional wave equation $u_t = c^2(u_{xx} + u_{yy})$ **(10)**

Q8   Solve the diffusion equation $u_t - ku_{xx} = e^{-x}$ on $x \in R, t > 0$ with initial condition $u(x, 0) =$ **(10)**
          0