Registration no: | | | | | | | | | | |

Total Number of Pages: 2

MSc.I
FMCE207

2nd Semester Regular/Back Examination– 2016-17
Math - II
BRANCH(S): M.Sc.I(AC)
Time: 3 Hour
Max marks: 70
Q Code:Z1063
Answer Question No.1 which is compulsory and any five from the rest
The figures in the right hand margin indicate marks

Q1 Answer the following questions: (2 x 10)

a) Give an example of a set which does not contain its lub and glb.

b) State Archimedean Principle.

c) What do you mean by absolute convergence of a sequence? Does absolute convergence of a sequence imply convergence of that sequence?

d) Examine whether the sequence (x_n) with $x_n = n^2 + n$ is convergent.

e) Examine whether the series $\sum_{n=1}^{\infty} \left(\frac{2n}{5^n}\right)$ is convergent.

f) Show that a group containing two elements must be abelian.

g) Let G be an abelian group and H any subgroup of G. Show that H is a normal subgroup of G.

h) Let G be a group and ϕ be a homomorphism on G, then show that $\left(\Phi(x)\right)^{-1} = \Phi(x^{-1})$, for each $x \in G$.

i) Define an integral domain.

j) Let R be a ring, then show that for all $a, b \in R, a0 = 0a = 0$.

Q2 a) Show that Q is an Archimedean ordered field which is not complete. [5]

b) State and prove Bolzano – Weierstrass Theorem for sets. [5]

Q3 a) Show that every convergent sequence is bounded. [5]

b) Show that the sequence (x_n) with $x_n = \left(1 + \frac{1}{n}\right)^n$ is convergent. [5]

Q4	a)	State and prove D' Alembert's Ratio Test on convergence of an infinite series.	[5]
	b)	Discuss the convergence of the series $\sum_{n=1}^{\infty}\left(\frac{1}{2^n}+\frac{1}{3^n}\right)$.	[5]
Q5	a)	If H is a subgroup of a group G, and $a \in G$. Let $aHa^{-1} = \{aha^{-1}: h \in H\}$. Show that aHa^{-1} is a subgroup of G.	[5]
	b)	Let H and K be two subgroups of a group G. Let $HK = \{hk: h \in H, k \in K\}$. Show that HK is a subgroup of G if and only if $HK = KH$.	[5]
Q6	a)	Let G and $\bar{G}$ be two groups and ϕ be a homomorphism of G into $\bar{G}$ with kernel K, then show that K is a normal subgroup of G.	[5]
	b)	Let G and $\bar{G}$ be two groups and ϕ be a homomorphism of G onto $\bar{G}$ with kernel K, then show that G/K is isomorphic to $\bar{G}$.	[5]
Q7	a)	Show that a finite integral domain is a field.	[5]
	b)	If p is a prime number then show that J_p the ring of integers mod p is a field.	[5]
Q8	a)	Show that a monotonic increasing sequence which is bounded above is convergent.	[3]
	b)	Show that the series $\sum_{n=1}^{\infty}\left(\frac{1}{n^p}\right)$ converges if $p > 1$.	[3]
	c)	Show that every permutation is the product of its cycles.	[4]

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