Registration no: | | | | | | | | | | |

Total Number of Pages: 2 **MSc.I**
FMCC202

2nd Semester Regular Examination– 2016-17
ANALYSIS-I
BRANCH(S): Mathematics and Computing
Time: 3 Hour
Max Marks: 70
Q Code : Z789

Answer Question No.1 which is compulsory and any five from the rest
The figures in the right hand margin indicate marks

Q1 Answer the following questions: (2 x 10)

a) Give an example of a set which is bounded below but not bounded above.
b) Give an example of an open set which is not an interval.
c) State Dedekind's property.
d) Prove that $\lim(n^{1/n}) = 1$
e) Define Cauchy sequence.
f) Test the convergence of the series $\sum a_n$ where a_n is $\frac{1}{\log n}$.
g) Give an example of uniformly convergent series not absolutely convergent.
h) State intermediate value theorem.
i) Prove that a function which is uniformly continuous on an interval is continuous on that interval.
j) Give an example of a sequence of continuous functions converging to a discontinuous function.

Q2 a) Show that every open interval is an open set. [5]
b) Prove that the set of rational numbers in [0, 1] is countable [5]

Q3 a) Prove that the union of two closed sets is a closed set. [5]

b) Prove that a set is closed iff its complement is open. [5]

Q4 a) Prove that every bounded sequence with a unique limit point is convergent. [5]
b) Prove that every convergent sequence is bounded and has a unique limit. [5]

Q5 a) Show that every Cauchy sequence is bounded and every convergent sequence is Cauchy. [5]

b) Suppose that $a_n \geq a_{n+1} \geq 0$. Then show that the series $\sum a_n$ converges if and only if the series $\sum_{k=0}^{\infty} 2^k a_{2^k}$ converges. [5]

Q6 a) Show that $f_n(x) = n^2 x^n (1-x), x \in [0,1]$ converges point wise (but not uniformly) to a function which is continuous on [0, 1]. [5]

b) Prove that if $\sum f_k(x)$ and $\sum g_k(x)$ are uniformly convergent, then $\sum(f_k(x) + g_k(x))$ is uniformly convergent. [5]

Q7 a) Show that the series $\sum \frac{x^n \sin nx}{n^p}$ $(p > 0)$ converges uniformly on the interval $[-1, 1]$. [5]

b) Prove that a function which is continuous on a closed interval is also uniformly continuous on that interval. [5]

Q8 a) Prove that $f(x) = \sin\frac{1}{x}, x \neq 0$
$= 0, x = 0$
Is not uniformly continuous 0n $[0, \infty]$. [3]

b) Show that the series $\sum \frac{1}{n^p}$ converges , if $p > 1$, and diverges if $p \leq 1$. [4]

c) Prove that the union of an arbitrary family of open sets is open. [3]

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