Registration no:

Total Number of Pages: 02

M. Sc.
15MMCC104

1st Semester Back Examination – 2017-18
Abstract Algebra
Branch : M.Sc.(MH)
Time: 3 Hours
Max marks: 70
Q Code:B1052
Answer Question No.1 which is compulsory and any five from the rest.
The figures in the right hand margin indicate marks.

Q1 **Answer the following questions:** **(2 x 10)**

a) Define a symmetric group.

b) Group of order 9 is abelian, true or false ? Explain.

c) Give an example of a dihedral group.

d) $f = (2\ 4\ 7)$, $g = (3\ 9\ 6)$ find fog and define Transposition.

e) Define Simple Ring.

f) Let $G = \{1, i, -i, -1\}$. Be a set, $i = \sqrt{-1}$. Is G a group with respect to the binary operation.(Multiplication). Is it abelian ?

g) Let R be a ring such that $x^3 = x \text{ for all } x \in R$. Is R commutative ?

h) Define a splitting field . Give an example.

i) Is an irreducible element is prime? Explain.

j) If O(G)=15, $f: G \to G$

$f(a) = a^4$, given mapping is Automorphism or not?

Q2 a) Let G be a group. Prove that N is a normal subgroup of G iff $gNg^{-1} = N \text{ for every } g \in G$. **(5)**

b) State and Prove Second Isomorphism theorem. **(5)**

Q3 a) G is a simple group, $N \lhd G$.N is maximal in G iff $\frac{G}{N}$ is simple. **(5)**

b) Prove that every group is isomorphic to a subgroup of A(S) for some appropriate S. **(5)**

Q4 a) Define Unique Factorization domain. Prove that in a unique factorization domain every non zero non unit element as a product of irreducible element is unique. **(5)**

b) Define a Euclidean ring. Prove that an Euclidean ring possesses a unit element. **(5)**

Q5 a) Prove that any two Sylow P-subgroups of a finite group G are conjugate in G. **(5)**

b) If R is an unique factorization domain then prove that the product of two primitive polynomials in R[x] is again a primitive polynomial in R[x] **(5)**

Q6 a) Show that if O(G)=30, G is not simple. **(5)**

b) Find a basis of $Q\left(\sqrt{3},\sqrt{5}\right)$ over Q. **(5)**

Q7 a) Prove that a polynomial of degree n over a field can have at most n roots in any extension field. **(5)**

b) Let Q be the field of rational numbers. Let $f(x) = x^3 - 2$. Find the three roots of f(x). **(5)**

Q8 a) Find the degree of a minimal splitting field of $x^6 + 1$ over Q. **(5)**

b) If E/K is Galois and F, an extension of K, then [EF:F] divides [E:K]. **(5)**