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Total Number of Pages: 03

B.Tech
PEEI5402

8th Semester Regular / Back Examination 2016-17

OPTIMAL CONTROL

BRANCH(S): EE, EEE

Time: 3 Hours

Max Marks: 70

Q.CODE: Z405

**Answer Question No.1 which is compulsory and any five from the rest.
The figures in the right hand margin indicate marks.**

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Q1 Answer the following questions: (2 x 10)

- Write down the expression for performance index for a finite time output regulator problem
- What do you understand by the increment and the variation of a functional?
- Find the variation of the functional

$$v(x_1, x_2) = \int_0^2 x_1 e^{-x_2} dt$$

Using the lemma $\delta v = \frac{d}{d\alpha} v(x + \alpha \delta x) |_{\alpha=0}$

- What do you understand by transversality conditions?
- What is 'Optimal Policy' in Dynamic Programming?
- Write down the Algebraic Riccati Equation and also the expression for the Optimal Control law.
- Write down the three equations to be solved for solving the Optimal Control problem by Hamiltonian Method. What are the three equations called?
- Differentiate between LQR and LQG controllers.
- A dynamical system is modelled by the scalar differential equation

$$\dot{x} = ax + bu, \quad x(t_0) = x_0$$

with the associated cost index

$$J(t_0) = \frac{1}{2} f x^2(t_f) + \frac{1}{2} \int_{t_0}^{t_f} (q x^2(t) + r u^2(t)) dt$$

Form the Hamiltonian function $H(t, x, u, \frac{\partial J^*}{\partial x})$ for the problem and determine $\partial H / \partial u$.

- What do you understand by the H_∞ norm of a transfer function?

Q2 Find the points in the three-dimensional Euclidean space that extremise the function (10)

$$f(x_1, x_2, x_3) = x_1^2 + x_2^2 + x_3^2$$

and lie on the intersection of the surfaces

$$\begin{aligned} x_3 &= x_1 x_2 + 5 \\ x_1 + x_2 + x_3 &= 1 \end{aligned}$$

Q3 a) Find the optimal control u^* for the system **(7)**

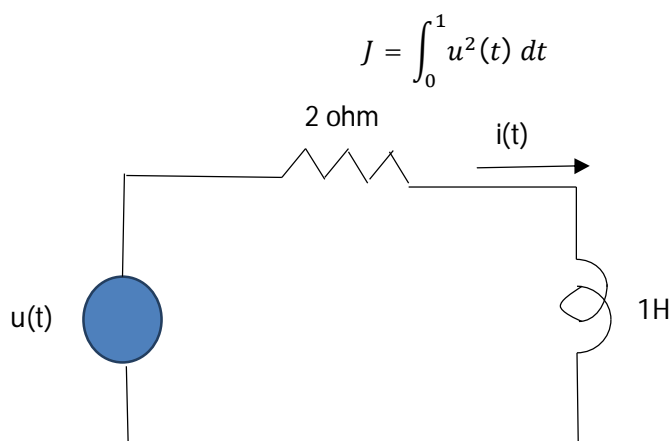
$$\dot{X} = \begin{bmatrix} 0 & 1 \\ -10 & 0 \end{bmatrix} X + \begin{bmatrix} 0 \\ 10 \end{bmatrix} u$$

Which minimises the performance index

$$J = \frac{1}{2} \int_0^2 u^2 dt \quad \text{Given } X(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad X(2) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

b) Write down the control equation, the state equation and the co-state equation for a system in terms of the Hamiltonian. **(3)**

Q4 a) For the circuit shown in figure, find the input voltage $u(t)$ that changes the current in the inductor from $i = 10A$ at $t = 0$ to $i = 0$ at $t = 1$ sec while minimizing the performance index **(6)**



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b) The performance index to be minimised for a linear quadratic regulator is given by **(4)**

$$J = \frac{1}{2} X(t_f)^T S X(t_f) + \int_{t_0}^{t_f} [X^T(t) Q X(t) + u^T(t) R u(t)] dt$$

Explain the significance of each of the three terms and the nature and usefulness of the matrices in the above expression.

Q5 Given, a second order plant **(10)**

$$\dot{x}_1(t) = x_2(t), \quad x_1(0) = 2$$

$$\dot{x}_2(t) = -2x_1(t) + x_2(t) + u(t), \quad x_2(0) = -3$$

And the performance index

$$J = \frac{1}{2} \int_0^\infty [2x_1^2(t) + 6x_1(t)x_2(t) + 5x_2^2(t) + u^2(t)] dt$$

Obtain the feedback control law using ARE.

Q6 a) Consider the following model of a dynamical system **(6)**

$$\dot{x} = 2x + u,$$

and the performance measure

$$J(x, u) = \int_0^{\infty} (x^2 + ru^2) dt$$

Find the value of r such that the optimal closed-loop system has pole at -3.

b) Discuss about Loop Transfer Recovery. **(5)**

Q7 Find the closed loop optimal control for the first order system **(10)**

$$\dot{x}(t) = -2x(t) + u(t)$$

With the performance index

$$J = \int_0^{\infty} [x^2(t) + u^2(t)] dt$$

Assume that $J^* = f x^2(t)$.

Q8 **Write short answer on any TWO:** **(5 x 2)**

- a)** Linear Fractional Transformation
- b)** Block diagram and Transfer function of LQG Controller
- c)** Discrete Ricatti Equation
- d)** Sub-Optimal Linear Regulator

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