

Sixth Semester Examination – 2008

DIGITAL SIGNAL PROCESSING

Full Marks – 70

Time : 3 Hours

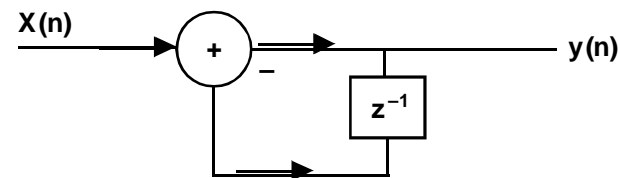
Answer either from Set-A or Set-B,
but not from both.

SET – A

Answer Question No. 1 which is compulsory
and any **five** from the rest.

The figures in the right-hand margin
indicate marks.

1. Answer the following questions : 2×10
- (a) Find the response of the system if $a = 1$,
 $b = -1$, $x(n) = \delta(n)$ and the system is
initially at rest.



- (b) Find out the Nyquist rate for the signal
 $x(t) = 25 \text{ COS } (500 \pi t)$.
- (c) What is the stability condition of an LTI
system ?
- (d) At which band an ideal filter is
distortionless ?
- (e) How the DFT and DTFT of one discrete
time signal related ?
- (f) Find out the impulse response of the LTI
system given by
 $y(n) = k_1 x(n) + k_2 x(n-1) + k_3 x(n-2)$.
- (g) What are the advantages of FFT over
DFT ?
- (h) Draw the signal flow graph of a first order
digital filter.
- (i) Show whether the systems are (i) Linear
/ Non linear, (ii) TV/TIV.

$$y(n) = \sum_{k=-\infty}^n x(k)$$

$$y(n) = x(n^2)$$

- (j) What is the aliasing effect ?

2. (a) Determine the impulse response for the given system described by difference equation. 6

$$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$$

- (b) Compute and sketch the step response of the system. 4

$$y(n) = \frac{1}{M} \sum_{k=0}^{N-1} x(n-k).$$

3. (a) Determine convolution of the following pairs of signal by means of ZT. 6

$$x_1(n) = 0.5^n u(n), \quad x_2(n) = \cos \pi n u(n).$$

- (b) Consider the Fir filter represented as $y(n) = x(n) + x(n-4)$. Compute and sketch the magnitude and phase spectrum. 4

4. (a) Let $x(n]$ be a real valued N point sequence. Develop a method to compute a N point DFT $x'(k)$, which contains only the odd harmonics by using a real $N/2$ point DFT. 5

- (b) Perform linear convolution of the following sequence by overlap add method. 5

$$x(n) = \{1, -1, 2, -2, 3, -3, 4, -4\}$$

$$h(n) = \{-1, 1\}.$$

5. $x(n) = \delta(n) + 2\delta(n-2) + \delta(n-3)$
 (i) Find the four point DFT of $x(n)$. 5

- (ii) If $y(n)$ is the four point circular convolution of $x(n)$ with itself, find $y(n)$ and four point DFT $Y(k)$. 5

6. Design an FIR digital filter approximating the ideal low frequency response.

$$H_d(\omega) = \begin{cases} 1 & |\omega| \leq \frac{\pi}{6} \\ 0 & \frac{\pi}{6} \leq |\omega| \leq \pi \end{cases}$$

- (i) Determine the coefficients of 25 tap filter based on window method with a rectangular window. 5

- (ii) Plot the magnitude and phase response of the filter. 5

7. (a) With impulse invariance, a first order pole in $H_a(s)$ at $s = s_k$ is mapped to a pole in $H(Z)$ at $Z = e^{s_k T}$.

$$\frac{1}{s - s_k} \Rightarrow \frac{1}{1 - e^{s_k T} z^{-1}}$$

- Determine how a second order pole is mapped with impulse invariance. 6

- (b) A second order continuous time filter has a system function

$$H(s) = \frac{1}{s-a} + \frac{1}{s-b}$$

Where $a < 0$ and $b < 0$ are real. Determine the locations of poles of $H(Z)$ if the filter designed using impulse invariance technique with $T = 2$ sec. 4

8. (a) Find the direct form II realization for the system described by difference equation. 6

$$Y(n) = \frac{3}{4}y(n-1) - \frac{3}{4}y(n-2) + x(n) - \frac{1}{3}x(n-1]$$

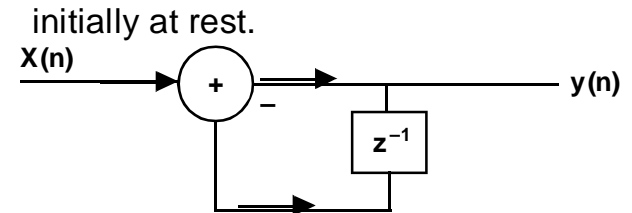
- (b) Explain the power spectrum estimation using the Bartlett method. 4

SET – B

Answer Question No. 1 which is compulsory and any **five** from the rest.

The figures in the right-hand margin indicate marks.

1. Answer the following questions : 2x10
 (a) Find the response of the system if $a = 1$, $b = -1$, $x(n) = \delta(n)$ and the system is



- (b) Find out the Nyquist rate for the signal $x(t) = 25 \cos(500 \pi t)$.
 (c) What is the stability condition of an LTI system ?
 (d) At which band an ideal filter is distortionless ?
 (e) How the DFT and DTFT of one discrete time signal related ?
 (f) Find out the impulse response of the LTI system given by
 $y(n) = k_1x(n) + k_2x(n-1) + k_3x(n-2)$.
 (g) What are the advantages of FFT over DFT ?
 (h) Draw the signal flow graph of a first order digital filter.
 (i) Show whether the systems are (i) Linear / Non linear, (ii) TV/TIV.

$$y(n) = \sum_{k=-\infty}^n x(k)$$

$$y(n) = x(n^2)$$

- (j) What is the aliasing effect ?

2. (a) Determine the impulse response for the given system described by difference equation. 6

$$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$$
 (b) Compute and sketch the step response of the system. 4

$$y(n) = \frac{1}{M} \sum_{k=0}^{N-1} x(n-k).$$

3. (a) Find the direct form II realization for the system described by difference equation. 6

$$Y(n) = \frac{3}{4}y(n-1) - \frac{3}{4}y(n-2) + x(n) - \frac{1}{3}x(n-1)$$

- (b) Consider the Fir filter represented as $y(n) = x(n) + x(n-4)$. Compute and sketch the magnitude and phase spectrum. 4
4. (a) Let $x(n)$ be a real valued N point sequence. Develop a method to compute a N point DFT $x'(k)$, which contains only the odd harmonics by using a real N/2 point DFT. 5
- (b) Perform linear convolution of the following sequence by overlap add method. 5
 $x(n) = \{1, -1, 2, -2, 3, -3, 4, -4\}$
 $h(n) = \{-1, 1\}.$

5. $x(n) = \delta(n) + 2\delta(n-2) + \delta(n-3)$
 (i) Find the four point DFT of $x(n)$. 5
 (ii) If $y(n)$ is the four point circular convolution of $x(n)$ with itself, find $y(n)$ and four point DFT $Y(k)$. 5
6. Determine the mean and the autocorrelation of the sequence $x(n)$, which is the output of a ARMA (1, 1) process described by difference equation $x(n) = 0.5 x(n-1) + w(n) - w(n-1)$. 10
7. For zero mean, jointly Gaussian random variable X_1, X_2, X_3, X_4 it is known that
 $E(X_1 X_2 X_3 X_4) = E(X_1 X_2) E(X_3 X_4) + E(X_1 X_3) E(X_2 X_4) + E(X_1 X_4) E(X_2 X_3)$
 use this result to derive the mean square value of $r'_{xx}(m)$ and the variance which is
 $\text{Var} [r'_{xx}(m)] = E [|r'_{xx}(m)|^2] - E [r'_{xx}(m)]^2$ 10
8. Determine the coefficient $\{h(n)\}$ of a linear phase FIR of length $N = 15$ which has a symmetric unit sample response and a frequency response that

$$H_r = \sum_{k=0}^{15} H_k e^{j\frac{2\pi k n}{15}}, \quad \begin{matrix} k=0, 1, 2, 3 \\ k=4, 5, 6, 7 \end{matrix}$$

satisfies the condition. 10